Study guide for Final Exam

Final exam problems include both the lecture contents and homework problems.

1. Review study guide for quiz 08-11.

2. Section 16.3:

   Study the definition and examples (Figure 16.22) of 'connected domain' and 'simply connected domain' (p985).

   Study the relation between conservative fields and

   (a) 'Gradient Fields' (Theorem 2, p986).
   (b) 'Loop Property' (Theorem 3, p987)
   (c) 'Component test' (p988),

   Which of them are equivalent to 'Conservative Fields'?

   Which of them are equivalent to 'Conservative Fields' only on simply connected domains?

   Which of the implications '$\Longleftarrow$', '$\Longrightarrow$' remains valid even if the domain is not simply connected (review homework 13, problem 2 for counter examples)?

   Hint: Study page 1 of Lecture 26.

3. Section 16.3:

   Study how to determine whether a vector field is conservative.

   Hint: Study page 6 of Lecture 26.

4. Section 16.3:

   For given functions $M(x, y, z), N(x, y, z), P(x, y, z)$ satisfying the component test, how does one find the potential function (if it exists) by way of direct integration (Example 3)?

5. Section 16.4:

   Study and memorize definitions of

   $$\text{div}\boldsymbol{F}(x, y) = M_x(x, y) + N_y(x, y) \tag{1}$$

   and

   $$\text{curl}\boldsymbol{F}(x, y) \cdot \boldsymbol{k} = N_x(x, y) - M_y(x, y) \tag{2}$$

   To memorize, it is easier to start with

   $$\text{div}\boldsymbol{F}(x, y, z) = \nabla \cdot \boldsymbol{F}(x, y, z) = (\partial_x, \partial_y, \partial_z) \cdot \boldsymbol{F}(x, y, z) = M_x(x, y, z) + N_y(x, y, z) + P_z(x, y, z)$$

   and

   $$\text{curl}\boldsymbol{F}(x, y, z) = \nabla \times \boldsymbol{F}(x, y, z) = (\partial_x, \partial_y, \partial_z) \times \boldsymbol{F}(x, y, z) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \partial_x & \partial_y & \partial_z \\ M & N & P \end{vmatrix}$$

   and view (1) and (2) as special case of $\boldsymbol{F} = (M(x, y),\ N(x, y),\ 0)$.

6. Section 16.4:

   Study and memorize Green's Theorem both in tangential form and normal form (p1000).

   Is Theorem 4 applicable to Example 5 of Section 16.3 on $R = \{x^2 + y^2 < 1\}$? Which part went wrong?

7. Section 16.4:

   Study how to apply Green's Theorem when the natural domain of $\boldsymbol{F}$ is not simply connected. For example:as Exercise 16.4, problem 39 and its counter part in tangential form (such as Example 5 of Section 16.3).

8. Section 16.4:

   Study the proof of Green's Theorem both in tangential form and normal form on a simple domain such as the domain on page 7 of Lecture 27.

   Study how to reduce the double integral in the domain to line integrals on the boundary of the domain using Fundamental Theorem of Calculus in 1D.