Homework 14

1. Section 16.3: Part II: Problems 3, 28, 32(d)(find a potential function instead), 33(b), 34.

2. Section 16.3: Let $\boldsymbol{F} = \dfrac{x}{\sqrt{x^2+y^2}}\,\boldsymbol{i} + \dfrac{y}{\sqrt{x^2+y^2}}\,\boldsymbol{j} + f(z)\,\boldsymbol{k}$ and $\boldsymbol{G} = \dfrac{-y}{x^2+y^2}\,\boldsymbol{i} + \dfrac{x}{x^2+y^2}\,\boldsymbol{j} + g(z)\,\boldsymbol{k}$. where f and g are continuous functions.
   (a) Show that both $\boldsymbol{F}$ and $\boldsymbol{G}$ satisfy the component test.
   (b) The natural domain of both $\boldsymbol{F}$ and $\boldsymbol{G}$ is $\{(x,y,z),\ x^2+y^2 \neq 0\}$ (that is where $\boldsymbol{F}$ and $\boldsymbol{G}$ are defined). Show that $\boldsymbol{F}$ is conservative in this domain by finding its potential function.
   (c) Show that $\boldsymbol{G}$ is NOT conservative in this domain (see Example 5 on p990).

3. Section 16.3: If $\boldsymbol{F} = M(x,y)\boldsymbol{i} + N(x,y)\boldsymbol{j}$ satisfies the component test on $\{(x,y),\ x^2+y^2 \neq 0\}$, how do you determine whether $\boldsymbol{F}$ is conservative?

4. Section 16.3: Let $\boldsymbol{F} = \dfrac{x}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{i} + \dfrac{y}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{j} + \dfrac{z}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{k}$. What is the natural domain of $\boldsymbol{F}$? Show that $\boldsymbol{F}$ satisfies the component test in this domain. Is this domain simply connected? Is $\boldsymbol{F}$ conservative in this domain?

5. Section 16.4: Problems 10, 17, 19, 23, 27, 38, 39.

   Hints: In problem 17, a polar curve $r = f(\theta)$ can be parameterized by $x(\theta) = f(\theta)\cos\theta$, $y(\theta) = f(\theta)\sin\theta$.

   Problem 19 can be computed easier using Green's Theorem.

   On a circle $x^2+y^2 = a^2$, it is a useful tip to remember that $\boldsymbol{n} = \dfrac{(x,y)}{a}$, $ds = a\,d\theta$ and $\boldsymbol{n}\,ds = (x,y)\,d\theta$.

6. Section 16.4: Review the proof of Green's Theorem in tangential form in Lecture 27. Then do the same for Green's Theorem in normal form for the vector field $\boldsymbol{F} = (M(x,y), N(x,y))$ on R, where M, N and their partial derivatives are all continuous in R, the region illustrated in class. That is, R is bounded by $x = 0$, $y = 0$ and the curve $y = f(x)$, $0 \le x \le a$ with $f(0) = b$ and $f(a) = 0$, which at the same time can be described as $x = g(y)$, $0 \le y \le b$ with $g(0) = a$ and $g(b) = 0$. In the computation of the line integrals on the three portions of ∂R, pay attention to finding suitable parametrizations with correct orientation.