Homework 10

1. Section 14.10: Problems 3, 7, 9, 12.

   Hint for Problem 12: Derive and solve a 2x2 linear system of equations.

2. Section 14.10: Follow up on problem 12:

   Derive a formula for $\left(\dfrac{\partial u}{\partial x}\right)_y$ if $u = U(x, y, z, w)$, $f(x, y, z, w) = 0$ and $g(x, y, z, w) = 0$ provided $f_z g_w - f_w g_z \neq 0$.

   Hint: count the numbers of dependent and independent variables first. Then use the result in Problem 12 to find $\left(\dfrac{\partial w}{\partial x}\right)_y$ and $\left(\dfrac{\partial z}{\partial x}\right)_y$ first, and then $\left(\dfrac{\partial u}{\partial x}\right)_y$.

   You can first take a specific example such as $U(x, y, z, w) = x^2 + y^2 + z^2 + w^2$, $f(x, y, z, w) = x + y + z^2 + w^2$, $g(x, y, z, w) = x^2 + y^2 + z + w$. Then generalize the result to general U, f and g.

3. Section 14.10: This is an alternative method for Section 14.8, problem 23 that also provides information for finding <u>local</u> extremes:

   Compute the partial derivatives with constrained variables $\left(\dfrac{\partial w}{\partial x}\right)_y$ and $\left(\dfrac{\partial w}{\partial y}\right)_x$ if $w = x - 2y + 5z$ and $x^2 + y^2 + z^2 = 30$.

   (a) Find critical points of w under the constrain $x^2 + y^2 + z^2 = 30$ by solving for $(x_0, y_0, z_0 = z(x_0, y_0))$ on $x^2 + y^2 + z^2 = 30$ such that

   $$\left(\frac{\partial w}{\partial x}\right)_y (x_0, y_0, z_0) = 0, \quad \left(\frac{\partial w}{\partial y}\right)_x (x_0, y_0, z_0) = 0. \tag{0}$$

   (b) Determine whether each of the critical points is a local min, a local max, or a saddle point by 2nd derivative test.

   (c) If (your student ID number (mod) 3) is 1, then change (0) to

   $$\left(\frac{\partial w}{\partial y}\right)_z (x_0, y_0, z_0) = 0, \quad \left(\frac{\partial w}{\partial z}\right)_y (x_0, y_0, z_0) = 0, \tag{1}$$

   instead, and proceed with corresponding 2nd derivative test.

   (d) If (your student ID number (mod) 3) is 2, then change (0) to

   $$\left(\frac{\partial w}{\partial z}\right)_x (x_0, y_0, z_0) = 0, \quad \left(\frac{\partial w}{\partial x}\right)_z (x_0, y_0, z_0) = 0 \tag{2}$$

   instead, and proceed with corresponding 2nd derivative test.

4. Section 15.1: Problems 21, 33, 36.

5. Section 15.2: Problems 11, 19, 29, 35, 43, 47, 69.