Homework 03

1. Section 10.4: Problems 15, 16, 17, 27, 29, 31, 43, 45, 51, 61, 62.

   Hints for problem 61:

   (a) For simplicity, just take a fixed case $p = 1.5$, $q = 3$ and proceed. The same argument works for all $p > 1$, $q > 0$. The case $p > 1$, $q \le 0$ is easier (Why?).

   (b) Instead of evaluating $\lim_{n\to\infty} \frac{(\ln n)^3}{n^{1.5-1.25}}$, it is and easier to evaluate $\left(\lim_{n\to\infty} \frac{\ln n}{n^{\frac{1.5-1.25}{3}}} \right)^3$.

   Hint for problem 62:

   For simplicity, just take a fixed case $p = 0.5$, $q = -2$ and proceed. The same argument works for all $0 < p < 1$, $q < 0$. The case $0 < p < 1$, $q \ge 0$ is easier.

2. Section 10.5: Do as many odd numbered problems in problem 17-43 as time permits.

   Hint: In general, the ratio test applies if you see the factorial $(\cdots)!$ appearing in a_n.

3. Section 10.5: Problems 61, 65.

   Hint for problem 61: multiply by $2 * 4 * \cdots * 2n = 2^n n!$ both on the denominator and the numerator.

   Hint for problem 65: Ratio test will be inconclusive, but root test works.

4. Section 10.6: Problems 11, 25, 26, 28, 29, 30, 35, 39, 41, 49, 53.

   Remark: Take the following for granted (need not prove it):

   The sequences $a_n = \frac{\ln n}{n}$, $b_n = \frac{\tan^{-1} n}{n^2+1}$, $c_n = \frac{\ln n}{n - \ln n}$, $d_n = (\sqrt{n+1} - \sqrt{n})$ are all decreasing for $n > 3$.

5. Section 10.6: (Hard! homework problem for next week)

   Show that $\int_\pi^\infty \frac{\sin t}{t}\, dt$ converges conditionally.

   Hint: $\int_\pi^b = \int_\pi^{[b]} + \int_{[b]}^b$ where $[b]$ denotes the largest integer n such that $n\pi \le b$.

   Remark (need not show it): The same conclusion holds for $\int_0^\infty \frac{\sin t}{t} dt$.