

Brief solutions to selected problems in homework 14 (v02)

1. Section 8.3: Solutions, common mistakes and corrections:

8.3.27

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sqrt{1-\cos x}} dx$$

(501-2) $1-\cos x = 2\sin^2\left(\frac{x}{2}\right)$

$$= -\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1-\cos x}} d\cos x$$

$\star x \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right] \Rightarrow \sin x > 0$
 $\sqrt{1-\cos^2 x} = |\sin x| = \sin x$

$$\stackrel{\ominus}{=} -\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} \cdot \frac{1}{\sqrt{1-\cos x}} d\cos x$$

$$= -\frac{1}{\cancel{\sqrt{2}}} \cdot \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{1+\cos x} d(1+\cos x)$$

$$= -\frac{1}{\cancel{\sqrt{2}}} \cdot \frac{2}{3} \cdot (1+\cos x)^{\frac{3}{2}} \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \left(-\frac{\sqrt{2}}{3} + \frac{\sqrt{3}}{2}\right) \times \sqrt{2}$$

Figure 1: Solution to Section 8.3, problem 27

$$\int_0^{\frac{\pi}{6}} \sqrt{1+\sin x} dx, \sqrt{1+\sin x} \cdot \sqrt{\frac{1-\sin x}{1-\sin x}} = \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}}$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos x}{\sqrt{1-\sin x}} dx = \frac{|\cos x|}{\sqrt{1-\sin x}}$$

$\because x \in [0, \frac{\pi}{6}]$
 $\cos x > 0$

Let $u = 1-\sin x$, $du = -\cos x dx$

$x=0, u=1; x=\frac{\pi}{6}, u=\frac{1}{2}$

$$= \int_1^{\frac{1}{2}} \frac{-du}{\sqrt{u}} = 2 \left[\sqrt{u} \right]_{\frac{1}{2}}^1 = 2 - \sqrt{2}$$

Figure 2: Solution to Section 8.3, problem 28

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^4 x}{\sqrt{1-\sin x}} dx = \int_{\frac{1}{2}}^1 \frac{t(2-t) \cdot \sqrt{t(2-t)}}{\sqrt{t}} dt \quad (1)$$

Let $1 - \sin x = t$ $1 - 2 + t = u$

$\frac{dt}{dx} = -\cos x \Rightarrow \sin x = 1 - t$ $(1 - \sin x)(1 + \sin x)$
 $= \cos^2 x = t(2-t)$
 $\cos x = -\sqrt{t(2-t)}$

$$(1) = - \int_{\frac{1}{2}}^1 (2-t) \cdot \sqrt{t} dt = \int_{\frac{1}{2}}^1 2t\sqrt{t} dt - \int_{\frac{1}{2}}^1 t^2\sqrt{t} dt$$

(sol 2) $u = \frac{\pi}{2} - x$

$$\Rightarrow \int_{-\pi/3}^{-\pi/2} \frac{\sin^4 u}{1 - \cos u} du = \left(\frac{4}{5} u^{5/2} - \frac{2}{7} u^{7/2} \right) \Big|_{u=1}^{u=\frac{3}{2}}$$

$$= \frac{4}{5} \left(\frac{3}{2}\right)^{5/2} - \frac{2}{7} \left(\frac{3}{2}\right)^{7/2} - \frac{18}{35}$$

Use $1 - \cos u = 2 \sin^2 \frac{u}{2}$

(note: $u \in [-\frac{\pi}{2}, -\frac{\pi}{3}] \Rightarrow \sin \frac{u}{2} < 0$)

Figure 3: Solution to Section 8.3, problem 29

$$\int \frac{\cos^3 x}{\sqrt{1+\sin x}} dx = \int \frac{1 - \sin^2 x}{\sqrt{1+\sin x}} \cos x dx$$

$$= \int \frac{1 - \sin^2 x}{\sqrt{1+\sin x}} d(\sin x) \quad (\text{let } u = \sin x)$$

$$= \int \frac{1 - u^2}{\sqrt{1+u}} du$$

$$= \int \sqrt{1+u} du - \int u\sqrt{1+u} du \quad \left(\begin{array}{l} \text{let } a = \sqrt{1+u} \\ a^2 - 1 = u \\ 2a da = du \end{array} \right)$$

$$= \int 2a^2 da - \int (a^2 - 1) 2a da$$

$$= \frac{2}{3} a^3 - \frac{2}{5} a^5 + \frac{2}{3} a^3 + C = \frac{4}{3} (1 + \sin x)^{3/2} - \frac{2}{5} (1 + \sin x)^{5/2} + C$$

Figure 4: Solution to Section 8.3, problem 29.5

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$$\int \sec x \tan^2 x \, dx$$

$$= (\sec x) \tan x - \int (\sec x) (\sec^2 x) \, dx$$

$$\int \sec^3 x \, dx$$

$$= \tan x \sec x - \int \tan x (\tan) \sec x \, dx$$

$$= \tan x \sec x - \int (\sec^2 - 1) \sec x \, dx$$

$$= \tan x \sec x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$\Rightarrow \int \sec^3 x \, dx = \frac{1}{2} (\tan x \sec x + \int \sec x \, dx)$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

Figure 5: Solution to Section 8.3, problem 34

$$\int \sec x \tan^2 x \, dx = \int \sec x (\sec^2 x - 1) \, dx$$

$$= \int \sec^3 x \, dx - \int \sec x \, dx, \quad \int \sec^3 x \, dx = \int \sec x \, d \tan x$$

$$= \sec x \tan x - \int \tan^2 x \sec x \, dx = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$\Rightarrow \int \sec^3 x \, dx = \frac{\sec x \tan x}{2} + \frac{1}{2} \int \sec x \, dx$$

$$\Rightarrow \int \sec^3 x \, dx - \int \sec x \, dx = \frac{\sec x \tan x}{2} - \frac{1}{2} \int \sec x \, dx$$

$$= \frac{\sec x \tan x}{2} - \frac{1}{2} \ln |\sec x + \tan x| + C$$

Figure 6: Solution to Section 8.3, problem 34. Another method

$$\begin{aligned}
 & \int \sin^2 \theta \cos 3\theta d\theta \\
 &= \int \frac{(1 - \cos 2\theta)}{2} \cos 3\theta d\theta \\
 &= \frac{1}{2} \left[\int \cos 3\theta d\theta - \int \cos \theta \cos 3\theta d\theta \right] \\
 &= \frac{1}{2} \left[\frac{\sin 3\theta}{3} - \frac{1}{2} \int (\cos 4\theta + \cos(-2\theta)) d\theta \right] \\
 &= \frac{1}{2} \left[\frac{\sin 3\theta}{3} - \frac{1}{2} \left[\frac{\sin 4\theta}{4} + \frac{\sin(-2\theta)}{2} \right] \right] + C
 \end{aligned}$$

$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
 $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
 $\Rightarrow \cos \alpha \cos \beta = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}$

Figure 7: Solution to Section 8.3, problem 57

$$\begin{aligned}
 & \int \frac{\sec^3 x}{\tan x} dx = \int \frac{\sec^2 x \sec x}{\tan x} dx \\
 & \sec^2 x = 1 + \tan^2 x \\
 & \frac{(1 + \tan^2 x) \sec x}{\tan x} = \frac{\sec x}{\tan x} + \tan x \sec x \\
 & \int \frac{\sec x}{\tan x} dx + \int \tan x \sec x dx \\
 &= \int \csc x dx + \int \tan x \sec x dx \\
 &= \ln |\csc x - \cot x| + \sec x + C
 \end{aligned}$$

Figure 8: Solution to Section 8.3, problem 63

2. Section 8.4: Solutions, common mistakes and corrections:

take $x = \sec t$

$$\int \frac{dx}{x^2 \sqrt{x^2 - 1}} = \int \frac{\sec t \tan t dt}{\sec^2 t \cdot \tan t} = \int \cos t dt = \sin t + C$$

$$\sin t = \sqrt{1 - \cos^2 t} = \sqrt{1 - \frac{1}{x^2}} = \frac{\sqrt{x^2 - 1}}{x}$$

$x > 1 \Rightarrow t \in [0, \frac{\pi}{2}]$, $\sin t > 0$

$$\int \frac{dx}{x^2 \sqrt{x^2 - 1}} = \frac{\sqrt{x^2 - 1}}{x} + C$$

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Figure 9: Solution to Section 8.4, problem 13

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{4x^2 dx}{(1-x^2)^{\frac{3}{2}}}$$

let $x = \sin \theta$, $dx = \cos \theta d\theta$
 $x=0, \theta = \sin^{-1} 0 = 0$, $x = \frac{\sqrt{3}}{2}, \theta = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$$\star \int_0^{\frac{\pi}{3}} \frac{4 \sin^2 \theta}{\cos^3 \theta} \cdot \cos \theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{3}} \tan^2 \theta d\theta$$

$\star \theta \in [0, \frac{\pi}{3}] \Rightarrow \cos \theta = |\cos \theta| = (1 - \sin^2 \theta)^{\frac{1}{2}}$

$$= 4 \int_0^{\frac{\pi}{3}} (\sec^2 \theta - 1) d\theta$$

$$= 4 (\tan \theta - \theta) \Big|_0^{\frac{\pi}{3}} = 4\sqrt{3} - \frac{4\pi}{3} \checkmark$$

Figure 10: Solution to Section 8.4, problem 23

$$\begin{aligned}
 & \int_0^4 \frac{e^t dt}{\sqrt{e^{2t} + 9}} \\
 &= \int_0^4 \frac{de^t}{\sqrt{e^{2t} + 3^2}} \quad \text{let } x = e^t \\
 &= \int_1^4 \frac{dx}{\sqrt{x^2 + 3^2}} \quad \left\{ \begin{array}{l} \text{let } x = 3 \tan \theta \quad \theta = \tan^{-1}(\frac{x}{3}) \\ dx = 3 \sec^2 \theta d\theta \end{array} \right. \\
 &= \int_{\tan^{-1}(1/3)}^{\tan^{-1}(4/3)} \frac{3 \sec^2 \theta d\theta}{3 \sec \theta} = \ln |\tan \theta + \sec \theta| \Big|_{\tan^{-1}(1/3)}^{\tan^{-1}(4/3)} \\
 &= \ln 9 - \ln |1 + \sqrt{10}| \quad \# \\
 & \quad \sqrt{\sec^2 \theta} = \sec \theta
 \end{aligned}$$

Figure 11: Solution to Section 8.4, problem 35

$$\begin{aligned}
 & \int_{\frac{1}{12}}^{\frac{1}{4}} \frac{2 dt}{\sqrt{t} + 4t \sqrt{t}} \\
 &= \int_{\frac{1}{12}}^{\frac{1}{4}} \frac{2 dt}{\sqrt{t}(1 + 4t)} \quad u = \sqrt{t} \\
 &= \int_{\frac{\sqrt{3}}{6}}^{\frac{1}{2}} \frac{2 \times 2u du}{u(1 + 4u^2)} \quad u = \frac{\tan \theta}{2} \\
 &= 4 \left(\frac{1}{2} \tan^{-1}(2u) \right) \Big|_{\frac{\sqrt{3}}{6}}^{\frac{1}{2}} \\
 &= \frac{\pi}{6} \quad \checkmark
 \end{aligned}$$

Figure 12: Solution to Section 8.4, problem 37

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$$\int \sqrt{\frac{4-x}{x}} dx, \text{ let } \sqrt{x} = u \Rightarrow dx = 2u du$$

$$= \int \frac{\sqrt{4-u^2}}{u} \cdot 2u du = 2 \int \sqrt{4-u^2} du$$

$u = 2 \sin \theta \Rightarrow \theta = \sin^{-1} \frac{u}{2}$
 $0 \leq \theta \leq \frac{\pi}{2} \Rightarrow du = 2 \cos \theta d\theta$

$$= 2 \int \sqrt{4-4\sin^2 \theta} \cdot 2 \cos \theta d\theta = 8 \int \cos^2 \theta d\theta$$

$$= 8 \int \frac{1+\cos 2\theta}{2} d\theta = 4 \left(\theta + \frac{\sin 2\theta}{2} \right) + C$$

$$= 4 \left(\theta + \sin \theta \cos \theta \right) + C$$

$$= 4 \left(\sin^{-1} \frac{u}{2} + \frac{u}{2} \cos \left(\sin^{-1} \frac{u}{2} \right) \right) + C$$

$$= 4 \sin^{-1} \frac{u}{2} + 2u \cdot \frac{\sqrt{4-u^2}}{2} + C$$

$$= 4 \sin^{-1} \frac{\sqrt{x}}{2} + \sqrt{x} \sqrt{4-x} + C$$

$$= 4 \sin^{-1} \frac{\sqrt{x}}{2} + \sqrt{4x-x^2} + C$$

Figure 13: Solution to Section 8.4, problem 45

47

$$\int \sqrt{x} \sqrt{1-x} dx$$

$(\because x > 0) \quad x = \sin^2 \theta, \quad dx = 2 \sin \theta \cos \theta d\theta$

$$= \int |\sin \theta| |\cos \theta| \cdot 2 \sin \theta \cos \theta d\theta$$

Assume $\sin \theta > 0$

$$= \int 2 \sin^2 \theta \cos^2 \theta d\theta$$

$$= \int 2 \left(\frac{1-\cos 2\theta}{2} \right) \left(\frac{1+\cos 2\theta}{2} \right) d\theta$$

$$= \int \frac{1}{2} (1-\cos^2 2\theta) d\theta = \int \frac{1}{2} \left(1 - \frac{1+\cos 4\theta}{2} \right) d\theta$$

$$= \frac{1}{4} \theta - \frac{1}{16} \sin 4\theta + C$$

If $\sin \theta < 0, \quad (-\frac{\pi}{2} < \theta < 0)$
 $\Rightarrow \cos \theta > 0, \quad 2 \sin^2 \theta \Rightarrow -2 \sin^2 \theta$
 But $\theta = \sin^{-1}(\sqrt{x}) \rightarrow \sin^{-1}(-\sqrt{x})$
 \therefore 結論不變

Figure 14: Solution to Section 8.4, problem 47

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$
 $A = 4 \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx$
 $x = a \sin \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$
 $= 4 \int_0^{\frac{\pi}{2}} b \sqrt{\cos^2 \theta} \cdot a \cos \theta d\theta$
 $= 4 \int_0^{\frac{\pi}{2}} ab \cos^2 \theta d\theta = 4ab \int_0^{\frac{\pi}{2}} \frac{\cos 2\theta + 1}{2} d\theta$
 $= 4ab \left[\frac{\sin 2\theta}{4} + \frac{\theta}{2} \right]_0^{\frac{\pi}{2}} = ab\pi$

Figure 15: Solution to Section 8.4, problem 54

3. Section 8.5: Solutions, common mistakes and corrections:

(85)
 $\int \frac{1}{y^2+1} dy + \int \frac{2y}{(y^2+1)^2} dy$
 (23)
 $\int \frac{y^2+2y+1}{(y^2+1)^2} dy = \tan^{-1} y + \frac{d(y^2+1)}{(y^2+1)^2}$
 $= \tan^{-1} y - \frac{1}{y^2+1} + C$
 $\frac{y^2+2y+1}{(y^2+1)^2} = \frac{B_1 y + C_1}{y^2+1} + \frac{B_2 y + C_2}{(y^2+1)^2}$
 $\Rightarrow (B_1 y + C_1)(y^2+1) + B_2 y + C_2 = y^2+2y+1$
 $= y^2+2y+1 \Rightarrow \begin{cases} B_1=0 & C_1=1 \\ B_2=2 & C_2=0 \end{cases}$

Figure 16: Solution to Section 8.5, problem 23

$$\int \frac{x^2}{x^4-1} dx = \int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1} \right) dx$$

$$x^2 = (A+B+C)x^3 + (A-B+D)x^2 + (A+B-C)x + (A-B-D)$$

$$D = \frac{1}{2}, C = 0, B = -\frac{1}{4}, A = \frac{1}{4}$$

$$\int \frac{x^2}{x^4-1} dx = \int \left(\frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{\frac{1}{2}}{x^2+1} \right) dx$$

Let $u = x-1$, $du = dx$; Let $v = x+1$, $dv = dx$

$$= \frac{1}{4} \int \frac{du}{u} - \frac{1}{4} \int \frac{dv}{v} + \frac{1}{2} \tan^{-1} x + C$$

$$= \frac{1}{4} \ln|x-1| - \frac{1}{4} \ln|x+1| + \frac{1}{2} \tan^{-1} x + C$$

Figure 17: Solution to Section 8.5, problem 29. Remark: this method requires solving A , B , C , D from 4 linear equations.

8.5.32.

$$\left[\frac{A\theta+B}{\theta^2+1} + \frac{C\theta+D}{(\theta^2+1)^2} + \frac{E\theta+F}{(\theta^2+1)^3} \right] = \frac{(A+B)(\theta^2+1)^2 + (C+D)(\theta^2+1) + E\theta + F}{(\theta^2+1)^3}$$

$$= \frac{\theta^4 - 4\theta^3 + 2\theta^2 - 3\theta + 1}{(\theta^2+1)^3} = f(\theta)$$

$$\Rightarrow f(\theta) = (\theta^2+1)(\theta^2-4\theta+1) + \theta$$

$$= (\theta^2+1)(\theta^2+1-4\theta) + \theta$$

$$= \underbrace{(0\theta+1)(\theta^2+1)}_{A\theta+B} + \underbrace{(-4\theta+0)(\theta^2+1)}_{C\theta+D} + \underbrace{(\theta+0)}_{E\theta+F}$$

$$\Rightarrow A = \frac{1}{\theta^2+1} + \frac{4\theta}{(\theta^2+1)^2} + \frac{\theta}{(\theta^2+1)^3}$$

$$\int \Rightarrow \tan^{-1} \theta - 2(\theta^2+1)^{-1} - \frac{1}{4}(\theta^2+1)^{-2} + C$$

Figure 18: Solution to Section 8.5, problem 32

(35)

$$\int \frac{9x^3 - 3x + 1}{x^3 - x^2} dx$$

① $\frac{9x^3 - 3x + 1}{x^3 - x^2} = 9 + \frac{2}{x} + \frac{-1}{x^2} + \frac{7}{x-1}$

$$= \int 9 + \frac{2}{x} - \frac{1}{x^2} + \frac{7}{x-1} dx$$

$$= 9x + 2 \ln|x| + \frac{1}{x} + 7 \ln|x-1| + C$$

Figure 19: Solution to Section 8.5, problem 35

(39)

$$\int \frac{e^t}{e^{2t} + 3e^t + 2} dt \quad \begin{array}{l} \text{let } e^t = y \\ e^t dt = dy \end{array}$$

$$= \int \frac{dy}{y^2 + 3y + 2}$$

$$= \int \frac{-1}{(y+2)} + \frac{1}{(y+1)} dy = -\ln|y+2| + \ln|y+1| + C$$

$$= \ln \left| \frac{e^t + 1}{e^t + 2} \right| + C$$

Figure 20: Solution to Section 8.5, problem 39

45. $\int \frac{1}{x^{3/2} - x^{1/2}} dx$, $\sqrt{x} = u$
 $\Rightarrow \frac{1}{2} \frac{dx}{\sqrt{x}} = du$, $x = u^2$
 $= \int \frac{1}{\sqrt{x} (x - 1)} dx$
 $= 2 \int \frac{1}{u^2 - 1} du$
 $= \int \frac{1}{u-1} + \frac{1}{u+1} du = \ln|u-1| - \ln|u+1| + C$
 $= \ln \left| \frac{\sqrt{x}-1}{\sqrt{x}+1} \right| + C$

Figure 21: Solution to Section 8.5, problem 45

$\int \frac{\sqrt{x+1}}{x} dx$, $\sqrt{x+1} = u$
 $\Rightarrow x+1 = u^2$
 $\Rightarrow dx = 2u du$
 $= \int \frac{u}{u^2-1} \cdot 2u du$
 $= 2 \int \frac{u^2}{u^2-1} du = 2 \int \left(\frac{u^2-1}{u^2-1} + \frac{1}{u^2-1} \right) du$
 $= 2 \int \left(1 + \frac{1}{2} \left(\frac{1}{u-1} - \frac{1}{u+1} \right) \right) du = 2u + \ln \left| \frac{u-1}{u+1} \right|$
 $= 2\sqrt{x+1} + \ln \left| \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right| + C$

Figure 22: Solution to Section 8.5, problem 47

4. Chap 8: Additional and Advanced Exercises. Solutions, common mistakes and corrections:

$$\begin{aligned}
 &\text{let } z = \tan \frac{x}{2}, \sin x = \frac{2z}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2} \\
 &dx = \frac{2dz}{1+z^2} \\
 &\int \frac{dz}{\sin x \cos x} = \int \frac{2dz}{\frac{2z}{1+z^2} \cdot \frac{1-z^2}{1+z^2}} \\
 &= \int \frac{2dz}{\frac{2z(1-z^2)}{(1+z^2)^2}} \\
 &= 2 \int \frac{1+z^2}{z(1-z^2)} dz \\
 &= 2 \int \frac{1}{z(1-z^2)} dz \\
 &= 2 \int \frac{1}{z(1-z)(1+z)} dz \\
 &= 2 \int \frac{1}{z(1-z^2)} dz \\
 &= 2 \cdot \frac{1}{2\sqrt{2}} \ln \left(\frac{z+1\sqrt{2}}{z-1\sqrt{2}} \right) + C \\
 &= \frac{\sqrt{2}}{2} \ln \frac{\tan \frac{x}{2} + 1\sqrt{2}}{\tan \frac{x}{2} - 1\sqrt{2}} + C
 \end{aligned}$$

$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$

Figure 23: Solution to Chap 8: Additional and Advanced Exercises, problem 47

$$\begin{aligned}
 &(49) \quad \tan \frac{x}{2} = z \\
 &\int \sec x dx = \int \frac{dx}{\cos x} = \int \frac{\frac{2}{1+z^2} dz}{\frac{1-z^2}{1+z^2}} \\
 &= \int \frac{2}{1-z^2} dz = \ln \left| \frac{1+z}{1-z} \right| + C \\
 &= \ln \left| \frac{1+\tan \frac{x}{2}}{1-\tan \frac{x}{2}} \right| + C \\
 &= \ln \left| \frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right| + C \quad \text{上下同乘 } (\cos \frac{x}{2} + \sin \frac{x}{2}) \\
 &= \ln \left| \frac{(\cos \frac{x}{2} + \sin \frac{x}{2})^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} \right| + C \\
 &= \ln \left| \frac{1 + \sin x}{\cos x} \right| + C \\
 &= \ln | \sec x + \tan x | + C
 \end{aligned}$$

Figure 24: Solution to Chap 8: Additional and Advanced Exercises, problem 49