Brief solutions to selected problems in homework 11

1. Section 6.2: Solutions, common mistakes and corrections:

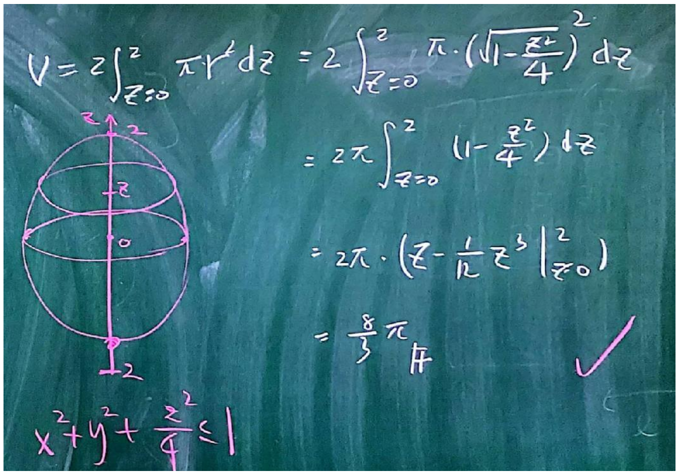


Figure 1: Solution to homework 11, problem 2

2. Section 6.3: Solutions, common mistakes and corrections:

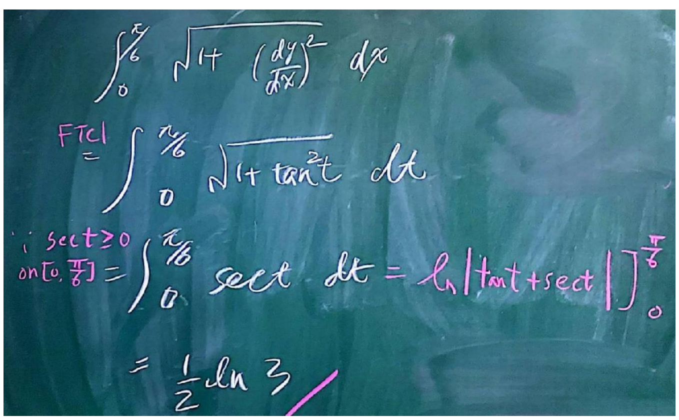


Figure 2: Solution to Section 6.3, problem 21

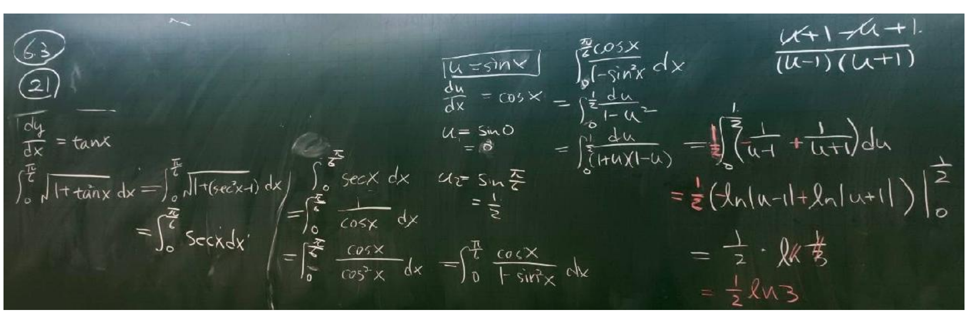


Figure 3: Solution to Section 6.3, problem 21, another method

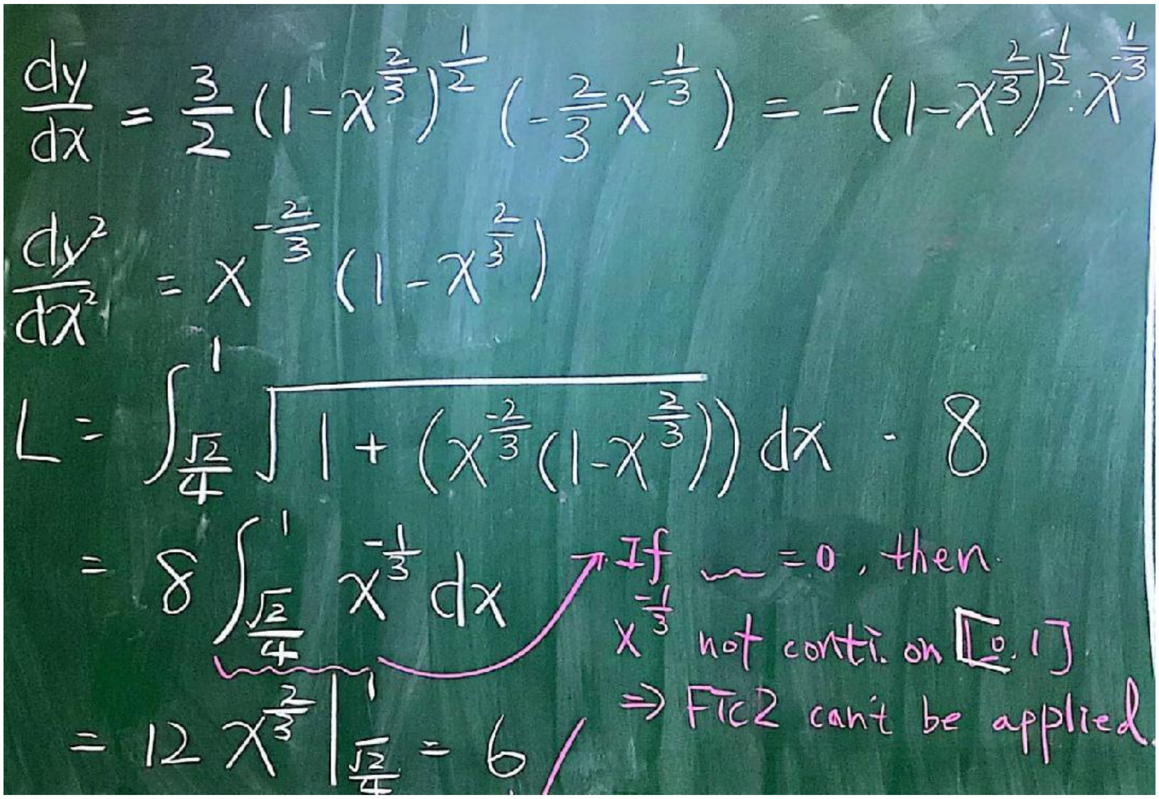


Figure 4: Solution to Section 6.3, problem 26

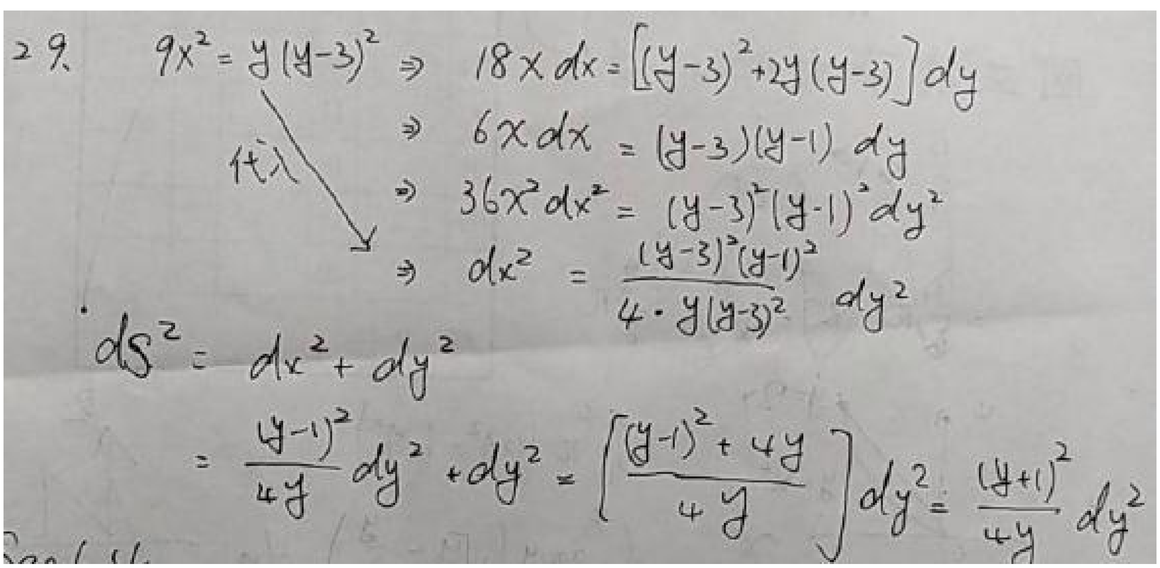


Figure 5: Solution to Section 6.3, problem 29

3. Section 6.4: Solutions, common mistakes and corrections:

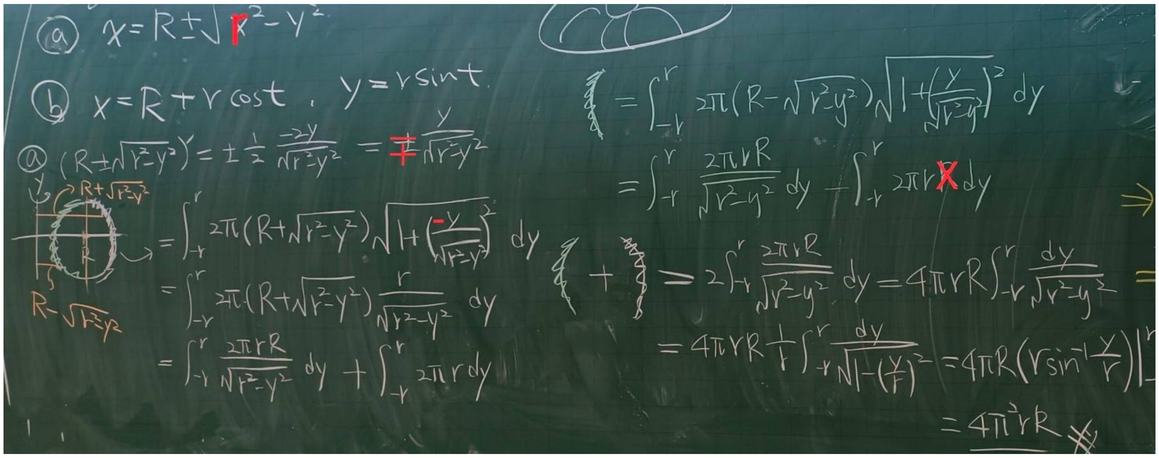


Figure 6: Solution to homework 11, problem 5(a)

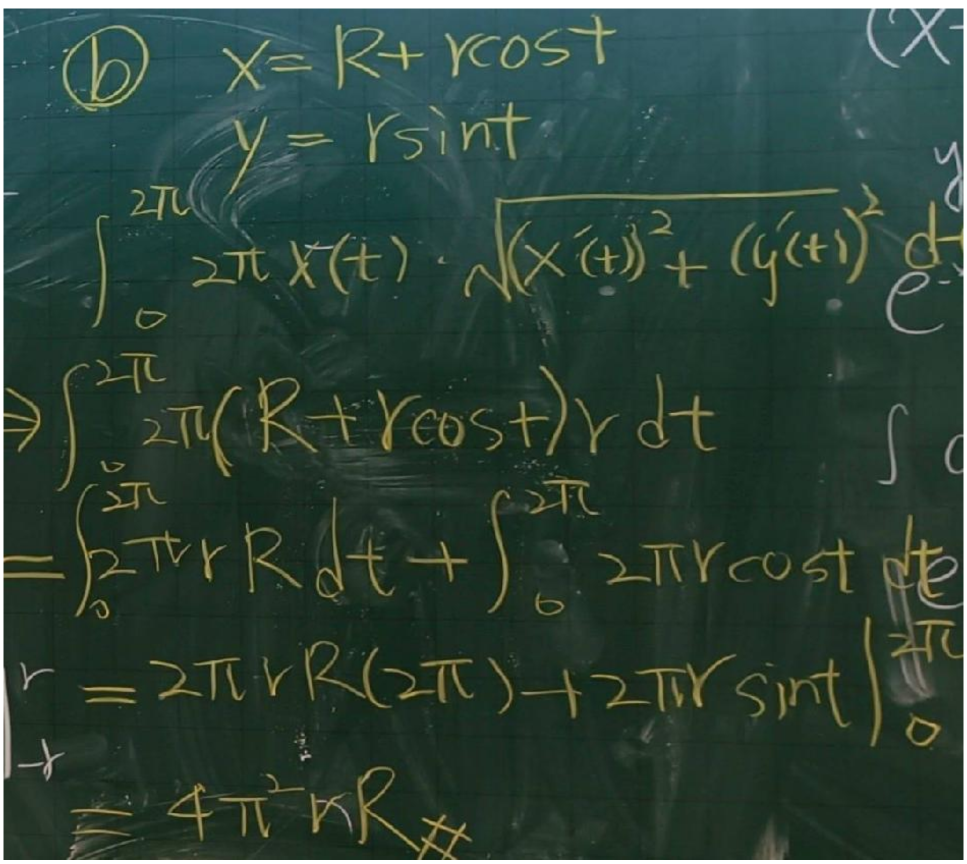
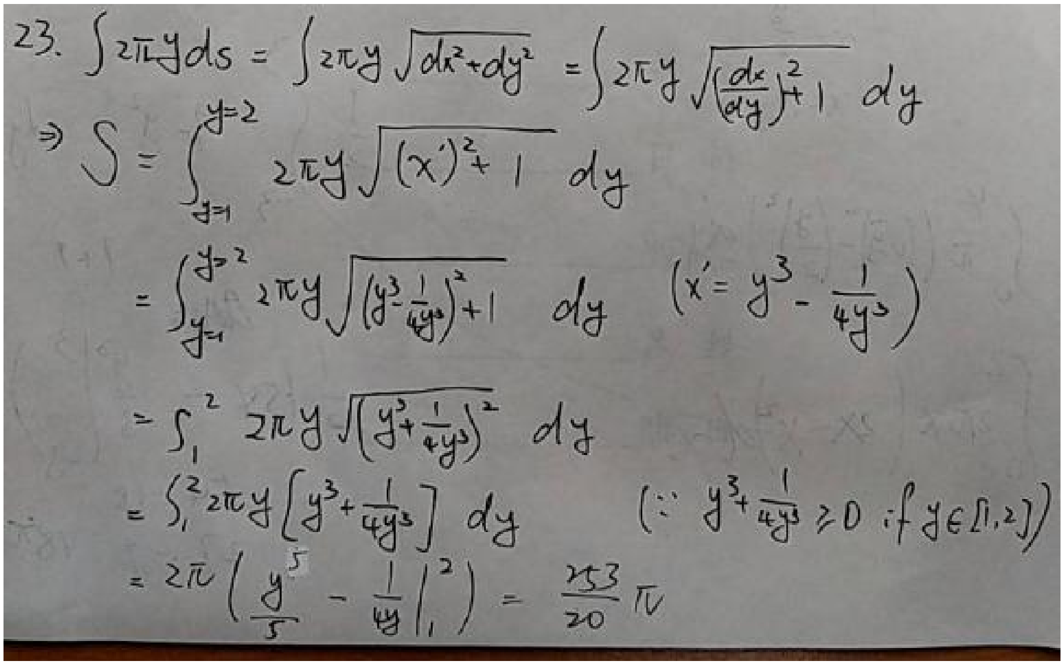


Figure 7: Solution to homework 11, problem 5(b)



23. $\int 2\pi y\, ds = \int 2\pi y \sqrt{dx^2+dy^2} = \int 2\pi y \sqrt{\left(\frac{dx}{dy}\right)^2+1}\, dy$

$\Rightarrow S = \int_{y=1}^{y=2} 2\pi y \sqrt{(x')^2+1}\, dy$

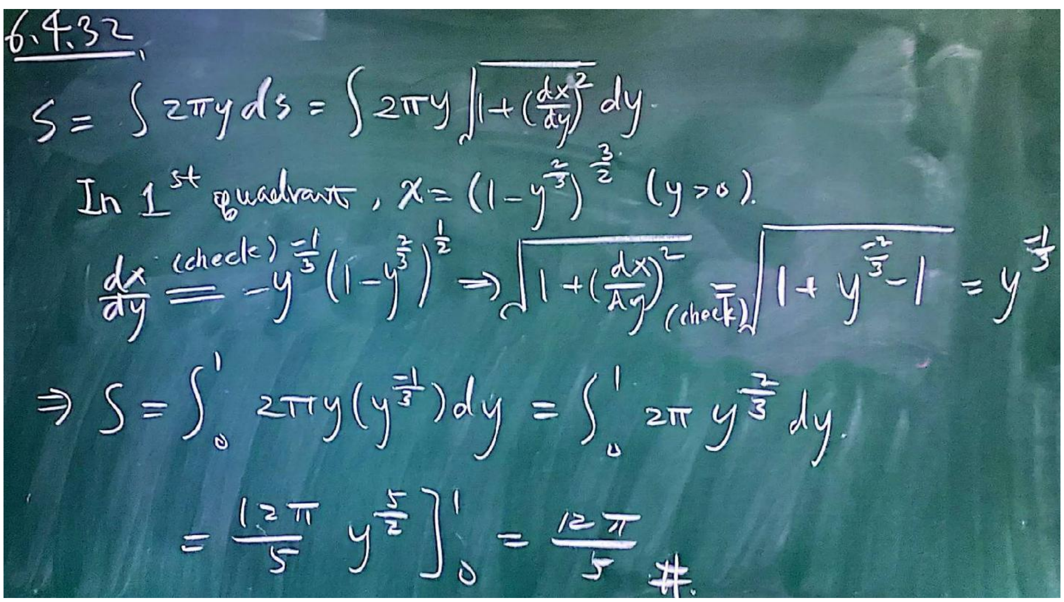
$= \int_{y=1}^{y=2} 2\pi y \sqrt{\left(y^3-\frac{1}{4y^3}\right)^2+1}\, dy \quad \left(x' = y^3 - \frac{1}{4y^3}\right)$

$= \int_1^2 2\pi y \sqrt{\left(y^3+\frac{1}{4y^3}\right)^2}\, dy$

$= \int_1^2 2\pi y \left[y^3+\frac{1}{4y^3}\right] dy \quad \left(\because y^3+\frac{1}{4y^3} \geq 0 \text{ if } y\in[1,2]\right)$

$= 2\pi\left(\frac{y^5}{5} - \frac{1}{4y}\Big|_1^2\right) = \frac{253}{20}\pi$

Figure 8: Solution to Section 6.4, problem 23



6.4.32

$S = \int 2\pi y\, ds = \int 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

In 1^{st} quadrant, $x = (1-y^{\frac{2}{3}})^{\frac{3}{2}}$ $(y>0)$.

$\frac{dx}{dy} \overset{(check)}{=\!=} -y^{\frac{-1}{3}}(1-y^{\frac{2}{3}})^{\frac{1}{2}} \Rightarrow \sqrt{1+\left(\frac{dx}{dy}\right)^2} \overset{(check)}{=} \sqrt{1+y^{\frac{-2}{3}}-1} = y^{\frac{-1}{3}}$

$\Rightarrow S = \int_0^1 2\pi y (y^{\frac{-1}{3}})\, dy = \int_0^1 2\pi y^{\frac{2}{3}}\, dy$

$= \frac{12\pi}{5} y^{\frac{5}{2}}\Big]_0^1 = \frac{12\pi}{5}$ #

Figure 9: Solution to Section 6.4, problem 32, method 1

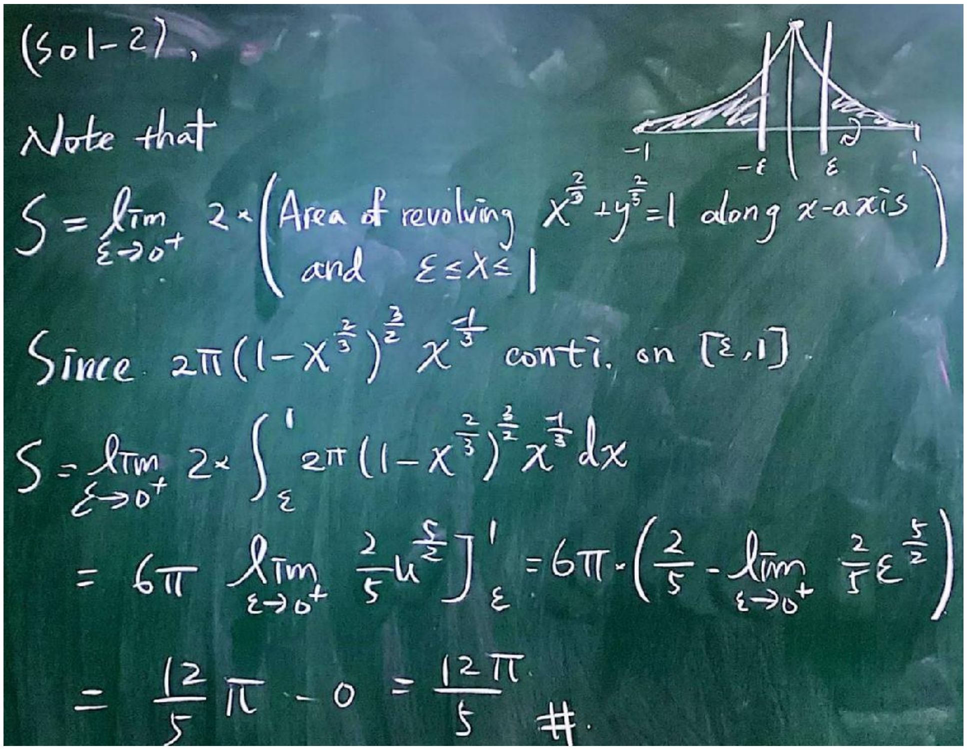


Figure 10: Solution to Section 6.4, problem 32, method 2

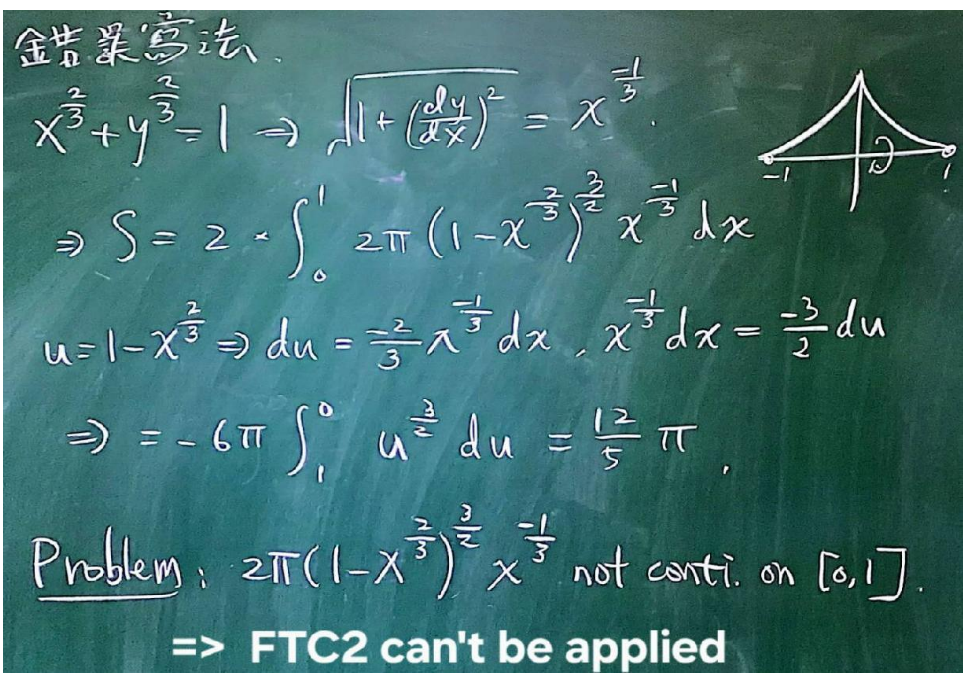


Figure 11: Solution to Section 6.4, problem 32, method 2, continued

4. Chapter 6, additional and advanced problems: Solutions, common mistakes and corrections:

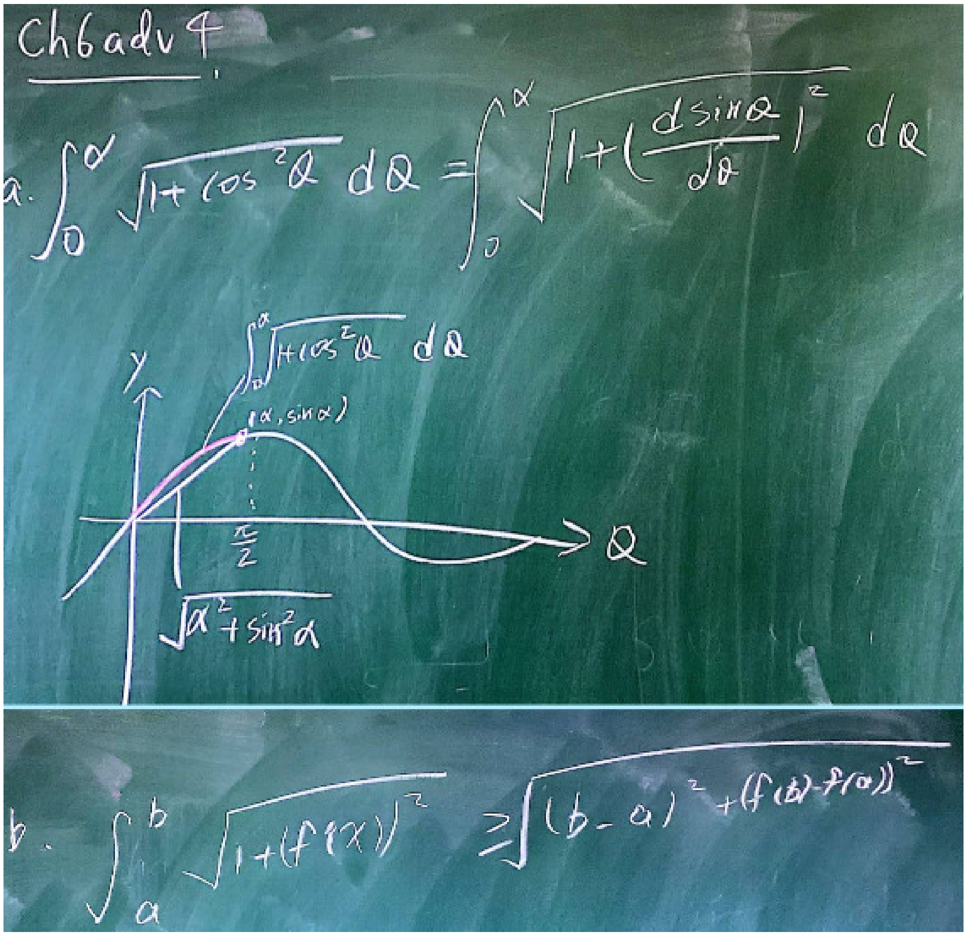


Figure 12: Solution to Chapter 6, additional and advanced problems: problem 4