

Brief solutions to selected problems in homework 10

1. Section 5.5: Solutions, common mistakes and corrections:

$\text{let } u = e^\theta$
 $du = d\theta \cdot e^\theta$
 $\int \frac{1}{\sqrt{e^{2\theta} - 1}} d\theta = \int \frac{1}{\sqrt{u^2 - 1}} \frac{du}{u}$
 $(u = e^\theta > 0) \Rightarrow \sec^{-1}(u) + C$
 $(\Rightarrow u = |u|)$
 $= \sec^{-1} e^\theta + C$

Figure 1: Solution to Section 5.5, problem 60

Remark: Another method for problem 60: multiply by $\frac{e^{-\theta}}{e^{-\theta}}$.

$\frac{d^2 y}{dx^2} = 4 \sec^2 x \tan^2 x, y(0) = 4, y'(0) = 1$
 $\int 4 \sec^2 x \tan^2 x dx, \text{ let } u = \tan^2 x, du = 2 \sec^2 x dx$
 $= \int 2u du = u^2 + C = \tan^2 x + C. y'(0) = 4 \Rightarrow C = 4$
 $\int (\tan^2 x + 4) dx = \int \sec^2 x dx - \int 1 dx + \int 4 dx$
 $= \frac{1}{2} \tan^2 x - x + 4x + C$
 $y(0) = -1 \Rightarrow C = -1$
 $\frac{1}{2} \tan^2 x + 3x - 1$

Figure 2: Solution to Section 5.5, problem 78

2. Section 5.6: Solutions, common mistakes and corrections:

$$\begin{aligned}
 & \int_0^1 r \sqrt{1-r^2} dr \quad u = 1-r^2 \\
 & = \frac{1}{2} \int_{\textcircled{1}=u(0)}^{\textcircled{0}=u(1)} u^{\frac{1}{2}} da \quad \frac{du}{dr} = -2r \\
 & = -\frac{1}{2} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}} \Big|_1^0 \\
 & = \frac{1}{3} \checkmark \quad (u=1-r^2, r dr = -\frac{1}{2} du.)
 \end{aligned}$$

Figure 3: Solution to Section 5.6, problem 2

$$\begin{aligned}
 & \int_{-1}^{-\frac{\sqrt{2}}{2}} \frac{1}{y \sqrt{4y^2-1}} dy \quad (u=2y) = \int_{-2}^{-\sqrt{2}} \frac{\cancel{2}}{\boxed{u} \sqrt{u^2-1}} \frac{1}{2} du \\
 & (\because u < 0) = \int_{-2}^{-\sqrt{2}} \frac{du}{\boxed{-|u|} \sqrt{u^2-1}} \\
 & = -\sec^{-1} u \Big|_{-2}^{-\sqrt{2}} = -\frac{\pi}{12} \#
 \end{aligned}$$

Figure 4: Solution to Section 5.6, problem 45

3. Chapter 5, additional and advanced problems: Solutions, common mistakes and corrections:

a. $\frac{d}{dx} [x \cos \pi x] = \cos \pi x - \pi x \sin \pi x$
 $2x f(x) = \cos \pi x - \pi x \sin \pi x$
 set $x^2 = y$
 $2\sqrt{y} f(y) = \cos \pi \sqrt{y} - \pi \sqrt{y} \sin \pi \sqrt{y}$
 $f(4) = \frac{\cos 2\pi - 2\pi \sin 2\pi}{4} = \frac{1}{4} \checkmark$

b. $\frac{[f(x)]^3}{3} = x \cos \pi x$, $f(x) = \sqrt[3]{3x \cos \pi x}$
 $f(4) = \sqrt[3]{12 \cos 4\pi} = \sqrt[3]{12} \checkmark$

Figure 5: Solution to Chapter 5, additional and advanced problems: problem 5

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right) \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{n+k} \right) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{1+\frac{k}{n}} \right) \frac{1}{n} \\ &= \int_0^1 \frac{1}{1+x} dx \\ &= \ln |1+x| \Big|_0^1 \\ &= \ln 2 \checkmark \end{aligned}$$

Figure 6: Solution to Chapter 5, additional and advanced problems: problem 21

$$\lim_{n \rightarrow \infty} \frac{1}{n} (e^{\frac{1}{n}} + e^{\frac{2}{n}} + \dots + e)$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n e^{\frac{k}{n}} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum f\left(\frac{k}{n}\right)$$

$$= \int_0^1 e^x dx = e^x \Big|_0^1 = e - 1$$

Figure 7: Solution to Chapter 5, additional and advanced problems: problem 22

4. Section 6.1: Solutions, common mistakes and corrections:

$$R(y) = b + \sqrt{a^2 - y^2}$$

$$r(y) = b - \sqrt{a^2 - y^2}$$

$$A(y) = \pi(R^2 - r^2) = \pi[(b + \sqrt{a^2 - y^2})^2 - (b - \sqrt{a^2 - y^2})^2]$$

$$= 4\pi b \sqrt{a^2 - y^2}$$

$$\int_{-a}^a (4\pi b \sqrt{a^2 - y^2}) dy = 4\pi b \cdot \frac{\pi a^2}{2}$$

$$= 2a^2 b \pi \quad \#$$

Figure 8: Solution to Section 6.1, problem 55

$$\begin{aligned}
 1^\circ V_1 &= \int_a^b \pi \cdot [f(x)]^2 dx = 4\pi \\
 \text{Diagram 1: A shaded region between } y=f(x) \text{ and } y=0 \text{ from } x=a \text{ to } x=b. \\
 2^\circ V_2 &= \pi \int_a^b \{ [f(x)+1]^2 - 1 \} dx \\
 \text{Diagram 2: A shaded region between } y=f(x)+1 \text{ and } y=1 \text{ from } x=a \text{ to } x=b. \\
 V_2 &= \pi \int_a^b [f(x)]^2 dx + 2\pi \int_a^b f(x) dx = 8\pi \\
 \therefore \int_a^b f(x) dx &= 2 \quad \# \quad \checkmark
 \end{aligned}$$

Figure 9: Solution to Section 6.1, problem 63

(63)

Find area $R = \int_a^b f(x) dx$

~~$y = -1$~~ correct answer

~~$\int_a^b \pi [f(x)+1]^2 dx = 8\pi$~~ $\times$ $\left(\int_a^b \pi (f(x)+1)^2 - 1^2 dx \right)$

$\Rightarrow \int_a^b [f(x)+1]^2 dx = 8$

$\Rightarrow \int_a^b (f(x)^2 + 2f(x) + 1) dx = 8$

$\Rightarrow \int_a^b f(x)^2 dx + 2 \int_a^b f(x) dx + \int_a^b 1 dx = 8$

$\Rightarrow 4 + 2 \int_a^b f(x) dx + (b-a) = 8$

$\Rightarrow \int_a^b f(x) dx = \frac{4-b+a}{2} = \text{Area } R \quad \#$

① X-axis:

$V = \int_a^b \pi [f(x)]^2 dx = 4\pi$

$\Rightarrow \int_a^b [f(x)]^2 dx = 4$

$\Rightarrow \int_a^b \pi (2f(x)) dx = 4\pi$

$\Rightarrow \int_a^b f(x) dx = 2$

$\Rightarrow \int_a^b f(x) dx = 2$

Figure 10: Solution to Section 6.1, problem 63, more details.

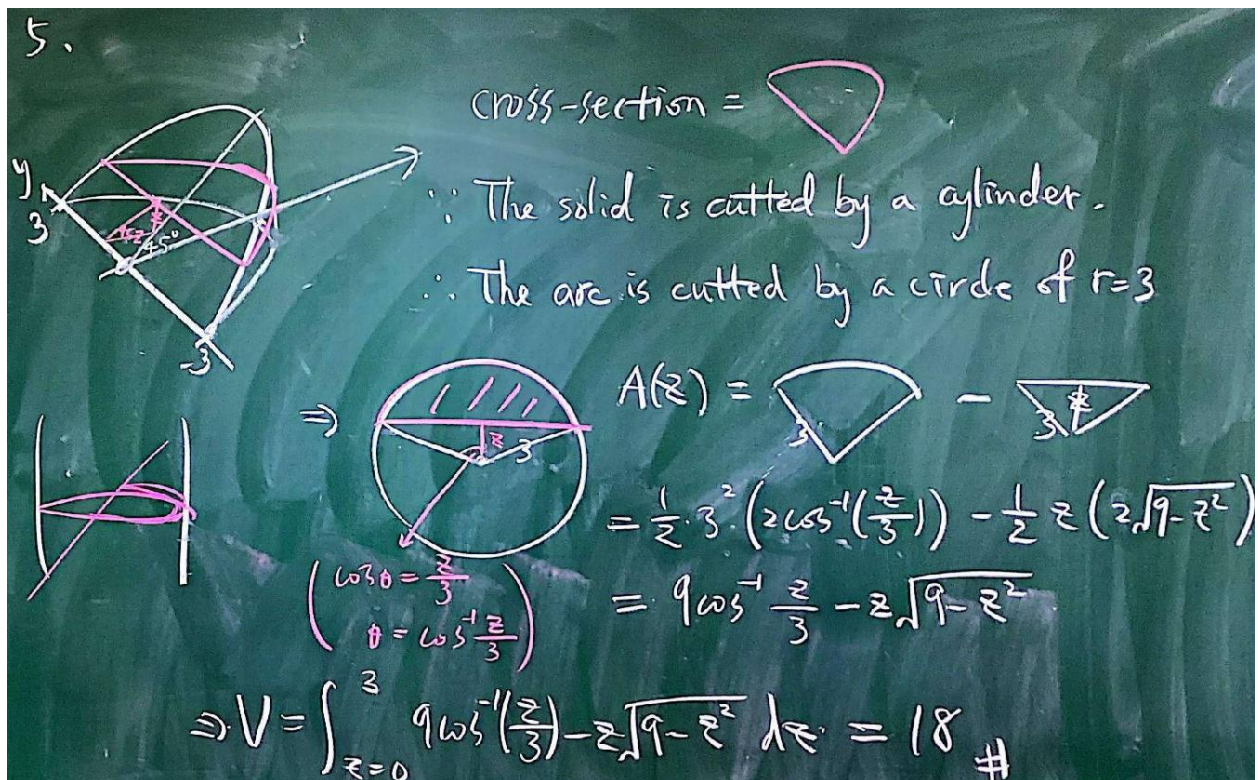


Figure 11: Solution to problem 5