

Brief solutions to selected problems in homework 08

1. Section 4.5: Solutions, common mistakes and corrections:

4.5.61,

$$\lim_{x \rightarrow \infty} \left(\frac{x+2}{x-1} \right)^x = \lim_{x \rightarrow \infty} \left(e^{\ln \frac{x+2}{x-1}} \right)^x = \lim_{x \rightarrow \infty} e^{x \ln \frac{x+2}{x-1}}$$

$$\therefore \lim_{x \rightarrow \infty} x \ln \left(\frac{x+2}{x-1} \right) (\infty \cdot 0) = \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{3}{x-1} \right)}{\frac{1}{x}} \quad \left(\frac{\infty}{\infty} \right)$$

$$\stackrel{(L'H)}{=} \lim_{x \rightarrow \infty} \frac{\frac{x-1}{x-1} \cdot \frac{-3}{(x-1)^2}}{-1 \cdot x^{-2}} = \lim_{x \rightarrow \infty} \frac{3x^2}{(x-1)^2} = 3$$

$$\therefore \lim_{x \rightarrow \infty} e^{x \ln \left(\frac{x+2}{x-1} \right)} = e^{\lim_{x \rightarrow \infty} x \ln \left(\frac{x+2}{x-1} \right)} = e^3$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x-1} \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} \right)^{x+1} \quad \checkmark$$

$$= \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} \right)^x \cdot \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} \right)^1 = e \cdot 1 = e^3 \quad \checkmark$$

Figure 1: Solution to Section 4.5, problem 61

4.5.65,

$$\lim_{x \rightarrow 0^+} x \tan \left(\frac{\pi}{2} - x \right)$$

3 $\tan \left(\frac{\pi}{2} - x \right) \rightarrow +\infty$
1 $x \rightarrow 0^+$
2 $\frac{\pi}{2} - x \rightarrow \frac{\pi}{2}^-$

$$= \lim_{x \rightarrow 0^+} \frac{x}{\cot \left(\frac{\pi}{2} - x \right)}$$

(0/0)

$$\stackrel{(0/0)}{=} \lim_{x \rightarrow 0^+} \frac{1}{\csc^2 \left(\frac{\pi}{2} - x \right)}$$

$$= \lim_{x \rightarrow 0^+} \sin^2 \left(\frac{\pi}{2} - x \right)$$

$$= 1 \quad \checkmark$$

Figure 2: Solution to Section 4.5, problem 65

$$\lim_{x \rightarrow 0} \frac{\tan 2x + ax}{x^3} = -b$$

the lim exists
 $\Rightarrow$ must be "0/0"
 $\Rightarrow a = -2$

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 2x + a}{3x^2} = \frac{8}{3}$$

$$\lim_{x \rightarrow 0} \frac{2}{3} \frac{\sin^2 2x}{x^2} \frac{1}{\cos^2 2x} = \frac{8}{3}$$

Figure 3: Solution to Section 4.5, problem 80

$$f(x) = e^{\ln f(x)} = e^{\frac{x \ln(1 + \frac{1}{x^2})}{1}}$$

84.10)
Let $f(x) = (1 + \frac{1}{x^2})^x \Rightarrow \ln f(x) = \frac{1}{x} \ln(1 + \frac{1}{x^2})$

$$\Rightarrow \lim_{x \rightarrow \infty} x \ln(1 + \frac{1}{x^2}) = \lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x^2})}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{1}{x^2}} \cdot \left(\frac{-2}{x^3}\right)}{-\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{x} \left(-\frac{1}{1 + \frac{1}{x^2}} \right) = 0$$

Figure 4: Solution to Section 4.5, problem 84(c)

4.5.86 (c)(d) logarithmic derivative

$$\ln(x^{\frac{1}{x^n}}) = \frac{\ln x}{x^n}, \quad \frac{d}{dx} \frac{\ln x}{x^n} = \frac{x^{n-1} - nx^{n-1} \ln x}{x^{2n}} = \frac{1-n \ln x}{x^{n+1}}$$

$$\left(\frac{d}{dx} (\ln y) = \frac{y'}{y} = \frac{1-n \ln x}{x^{n+1}} \Rightarrow y' = y \cdot \frac{1-n \ln x}{x^{n+1}} = x^{\frac{1}{x^n}} \frac{1-n \ln x}{x^{n+1}} \right)$$

critical pt: $y' = 0$ ($\because y' \text{ DNE} \Rightarrow x=0$ ~~$x=0$~~) $\Rightarrow n \ln x = 1, x = e^{\frac{1}{n}}$

$y' > 0 \Leftrightarrow 1-n \ln x > 0 \Leftrightarrow x < e^{\frac{1}{n}} \Rightarrow x = e^{\frac{1}{n}}$ local max

$y' < 0 \Leftrightarrow \dots \Leftrightarrow x > e^{\frac{1}{n}} \Rightarrow y = e^{\frac{1}{e^n}}$ abs. max

$$\ln \lim_{x \rightarrow \infty} x^{\frac{1}{x^n}} = \lim_{x \rightarrow \infty} \frac{\ln x}{x^n} \xrightarrow{\text{L'H}} \lim_{x \rightarrow \infty} \frac{1}{n x^{n-1}} = \frac{1}{n} \lim_{x \rightarrow \infty} \frac{1}{x^{n-1}} = 0$$

$$\lim_{x \rightarrow \infty} x^{\frac{1}{x^n}} = e^0 = 1 \quad \checkmark$$

Figure 5: Solution to Section 4.5, problem 84(c,d)

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{e^{\frac{1}{x^2}}} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{-\frac{1}{x^2}}{e^{\frac{1}{x^2}} \cdot (-\frac{2}{x^3})} = \lim_{x \rightarrow 0} \frac{x}{e^{\frac{1}{x^2}} \cdot 2} = 0$$

l'Hopital's rule

Figure 6: Solution to Section 4.5, problem 88

2. Section 4.6: Solutions, common mistakes and corrections:

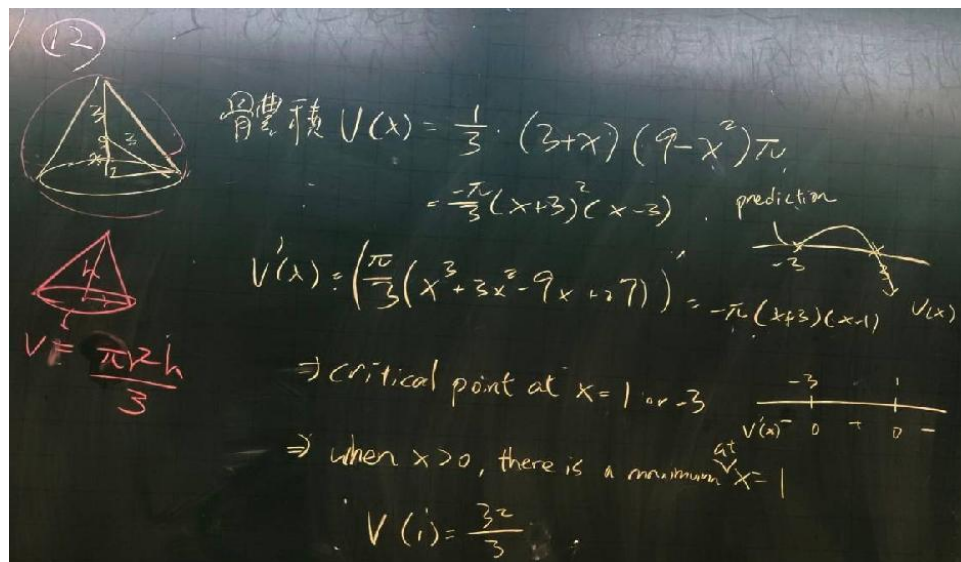


Figure 7: Solution to Section 4.6, problem 12

3. Section 4.8: Solutions, common mistakes and corrections:

$\Rightarrow$ 设 $H(x) = F(x) - g(x)$

$\frac{d}{dx} H(x) = F'(x) - g'(x) = 0$
 $= f(x) - f(x) = 0$

$\Rightarrow H(x) = C$

$H(x_0) = F(x_0) - g(x_0) = y_0 - y_0 = C$

$C=0 \Rightarrow F(x) = g(x)$

Figure 8: Solution to Section 4.8, problem 128