Brief solutions to selected problems in homework 03

1. Section 2.6: Solutions, common mistakes and corrections:

Sec 2.6

61. For any $M > 0$, take $\delta = \left(\frac{1}{M}\right)^{\frac{3}{2}} > 0$ s.t.

(a) if $0 < x < \delta$ then $f(x) \geq \frac{1}{x^{\frac{2}{3}}} > \frac{1}{\delta^{\frac{2}{3}}} = M$

(b) if $0 < -x < \delta$ then $f(x) \geq \frac{1}{x^{\frac{2}{3}}} > \frac{1}{\delta^{\frac{2}{3}}} = M$

(c) if $0 < x-1 < \delta$ then $f(x) \geq \frac{2}{(x-1)^{\frac{2}{3}}} > \frac{2}{\delta^{\frac{2}{3}}} = 2M > M$

(d) if $0 < 1-x < \delta$ then $f(x) \geq \frac{2}{(x-1)^{\frac{2}{3}}} > \frac{2}{\delta^{\frac{2}{3}}} = 2M > M$

85. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+3x} - \sqrt{x^2-2x}\right) = \lim\limits_{x\to\infty} \frac{5x}{\sqrt{x^2+3x} + \sqrt{x^2-2x}}$

$= \lim\limits_{x\to\infty} \frac{5}{\sqrt{1+\frac{3}{x}} + \sqrt{1+\frac{2}{x}}}$

$\left(\lim\limits_{x\to\infty} \frac{1}{x} = 0\right) = \frac{5}{2}$

92. For any $M > 0$, take $\delta = \sqrt{\frac{1}{M}} > 0$,

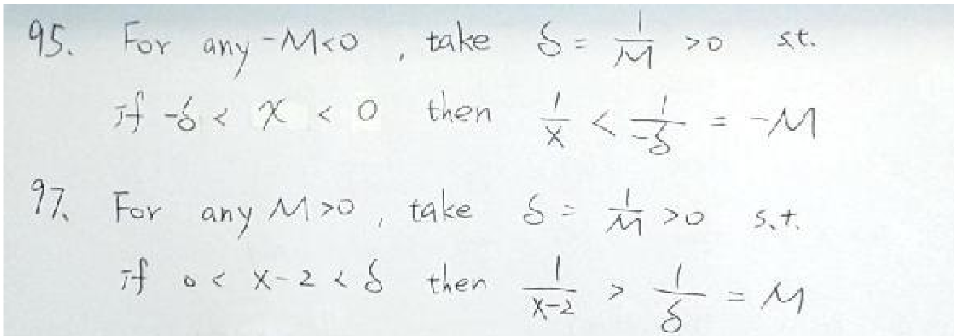
$0 < |x+5| < \delta \Rightarrow 0 < |x+5|^2 < \delta^2 = \frac{1}{M} \Rightarrow \frac{1}{(x+5)^2} > M$

93. (a) $\lim\limits_{x\to c^-} f(x) = \infty \Leftrightarrow$ For any $M > 0$, there exists a $\delta > 0$, s.t. if $-\delta < x-c < 0$ then $f(x) > M$

(b) $\lim\limits_{x\to c^+} f(x) = -\infty \Leftrightarrow$ For any $-M < 0$, there exists a $\delta > 0$ s.t. if $0 < x-c < \delta$ then $f(x) < -M$

(c) $\lim\limits_{x\to c^-} f(x) = -\infty \Leftrightarrow$ For any $-M < 0$, there exists a $\delta > 0$ s.t. if $-\delta < x-c < 0$ then $f(x) < -M$

Figure 1: Solution to selected problems in Section 2.6, part 1



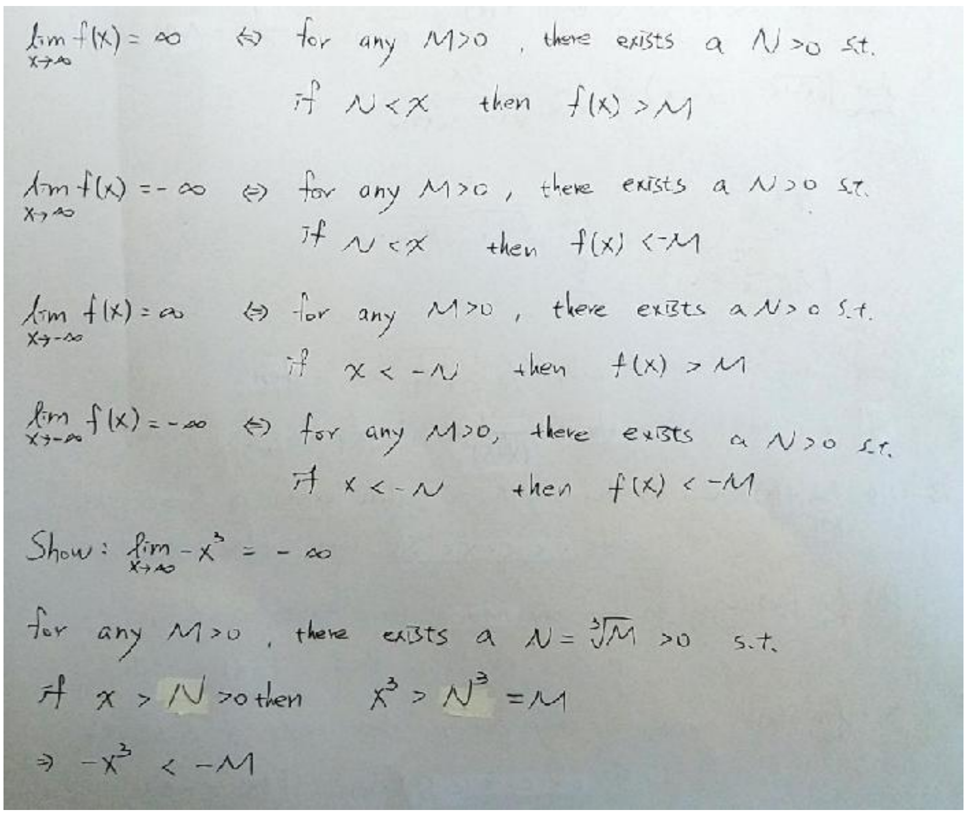
95. For any $-M<0$, take $\delta = \frac{1}{M} > 0$ s.t.

if $-\delta < x < 0$ then $\frac{1}{x} < \frac{1}{-\delta} = -M$

97. For any $M>0$, take $\delta = \frac{1}{M} > 0$ s.t.

if $0 < x-2 < \delta$ then $\frac{1}{x-2} > \frac{1}{\delta} = M$

Figure 2: Solution to selected problems in Section 2.6, part 2



$\lim_{x\to\infty} f(x) = \infty$ $\Leftrightarrow$ for any $M>0$, there exists a $N>0$ s.t.
if $N<x$ then $f(x)>M$

$\lim_{x\to\infty} f(x) = -\infty$ $\Leftrightarrow$ for any $M>0$, there exists a $N>0$ s.t.
if $N<x$ then $f(x)<-M$

$\lim_{x\to-\infty} f(x) = \infty$ $\Leftrightarrow$ for any $M>0$, there exists a $N>0$ s.t.
if $x<-N$ then $f(x)>M$

$\lim_{x\to-\infty} f(x) = -\infty$ $\Leftrightarrow$ for any $M>0$, there exists a $N>0$ s.t.
if $x<-N$ then $f(x)<-M$

Show: $\lim_{x\to\infty} -x^3 = -\infty$

for any $M>0$, there exists a $N=\sqrt[3]{M}>0$ s.t.

if $x > N > 0$ then $x^3 > N^3 = M$

$\Rightarrow -x^3 < -M$

Figure 3: Solution to Homework 03, problem 2

2. Section 3.2: Solutions, common mistakes and corrections:

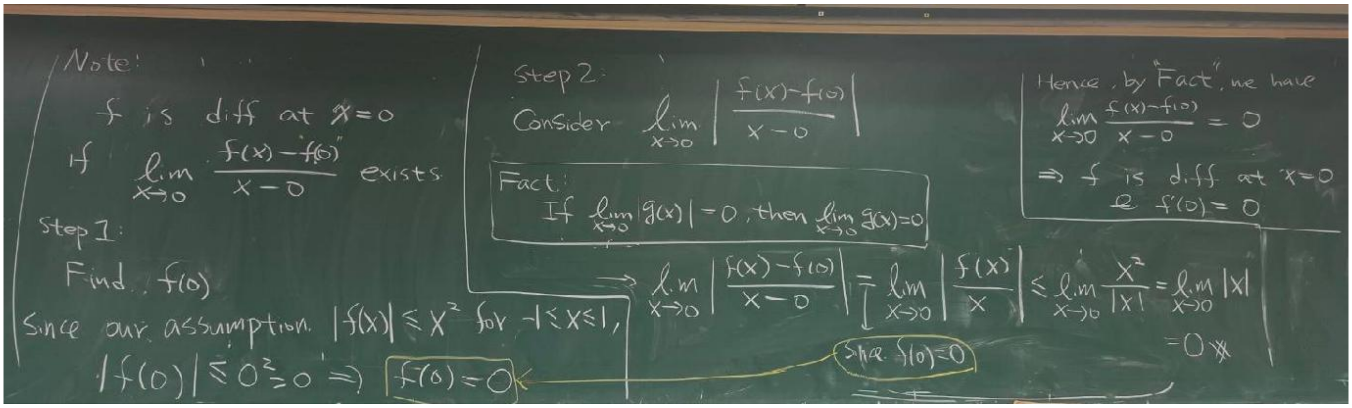


Figure 4: Solution to Section 3.2, problem 58(a). Step 1: show that $f(0) = 0$. Step 2: show that $f'(0) = 0$ as in problem 58(b)

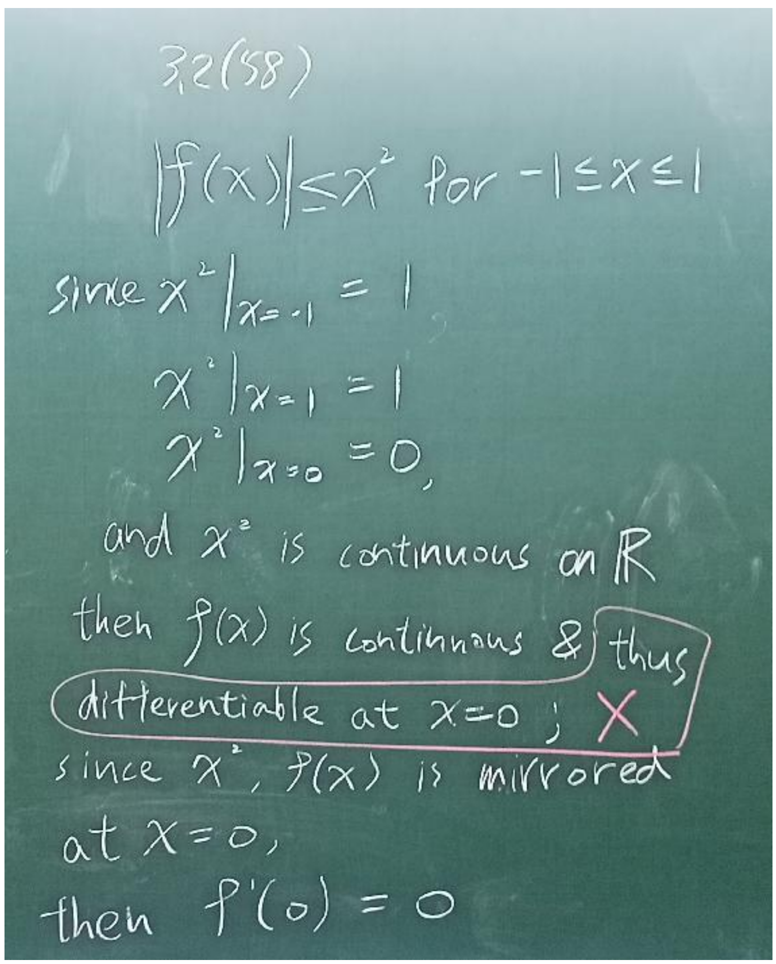


Figure 5: Common mistakes to Section 3.2, problem 58(a)

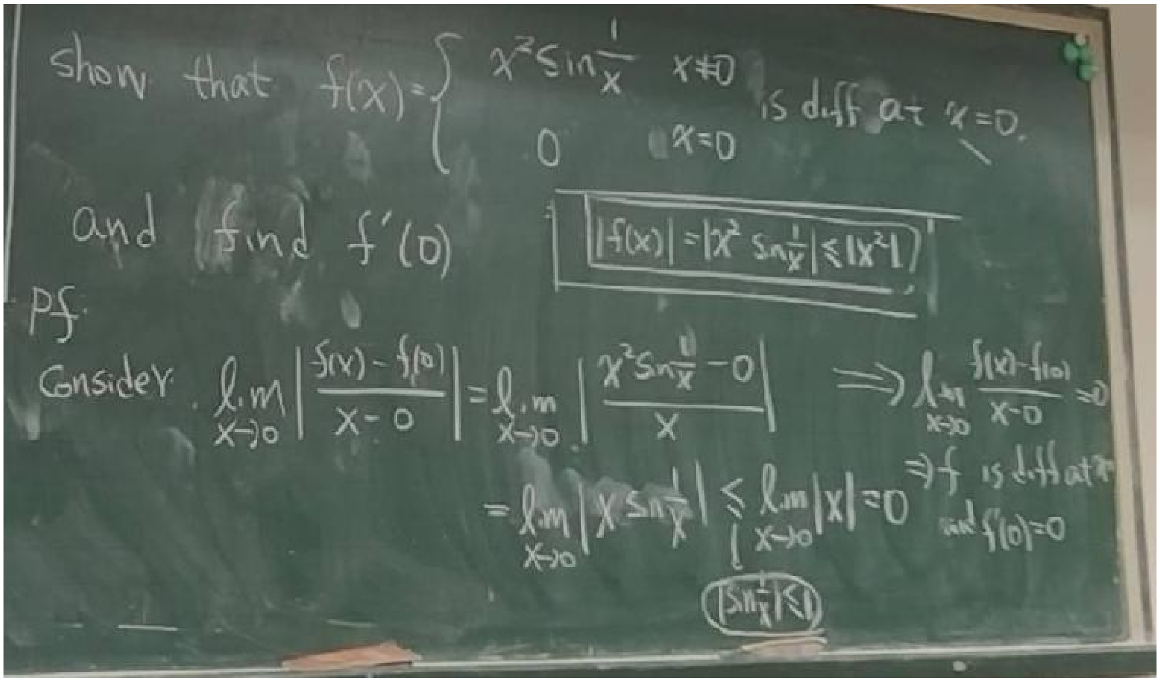


Figure 6: Solution to Section 3.2, problem 58(b)

3. Section 3.3: Solutions, common mistakes and corrections:

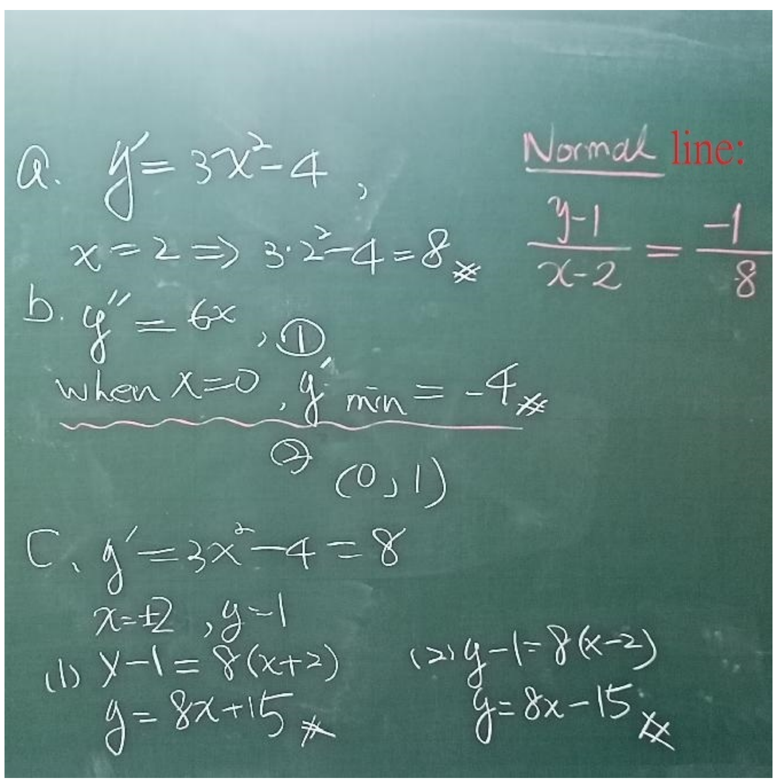


Figure 7: Solution to Section 3.3, problem 55

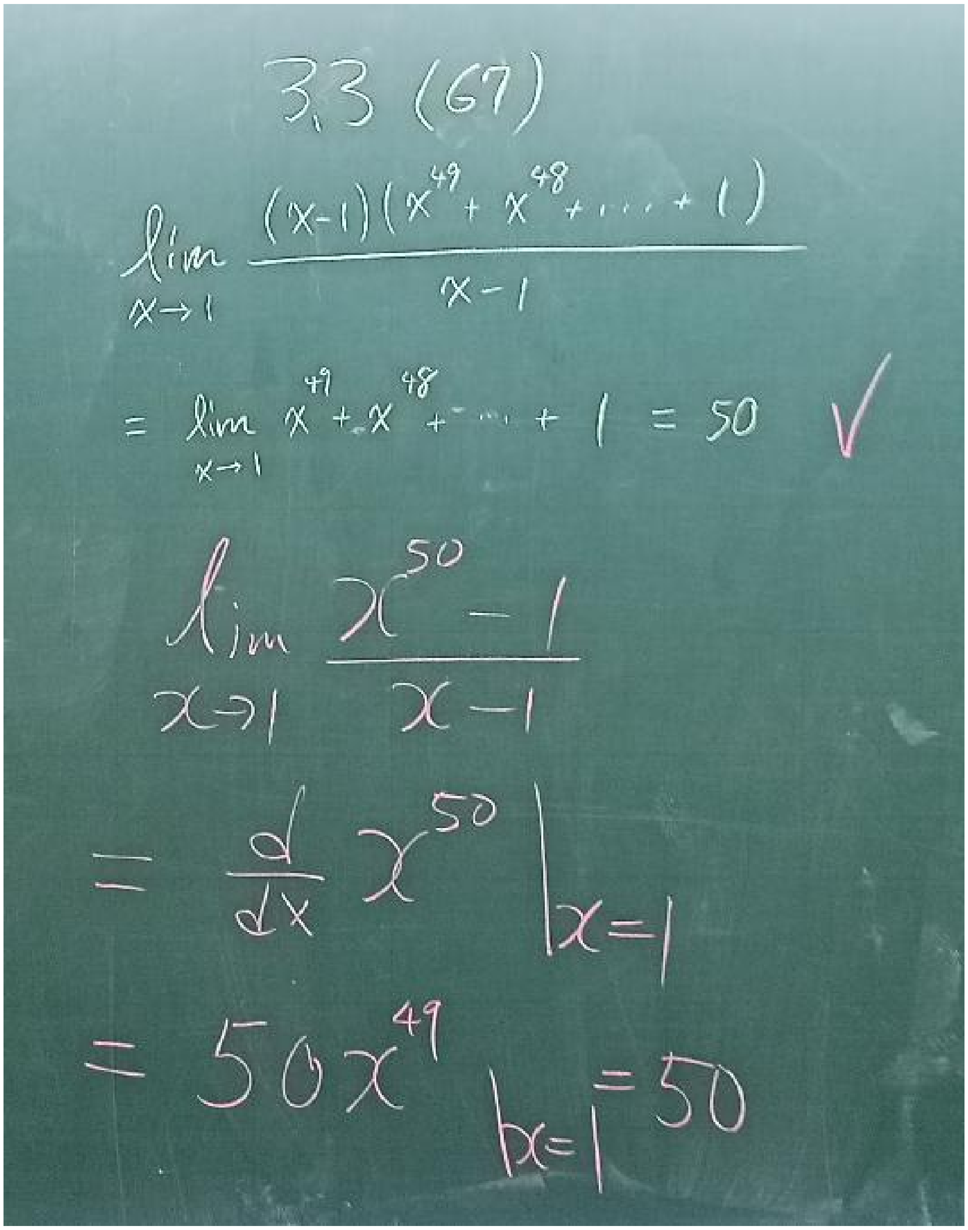


Figure 8: A trick for Section 3.3, problem 67

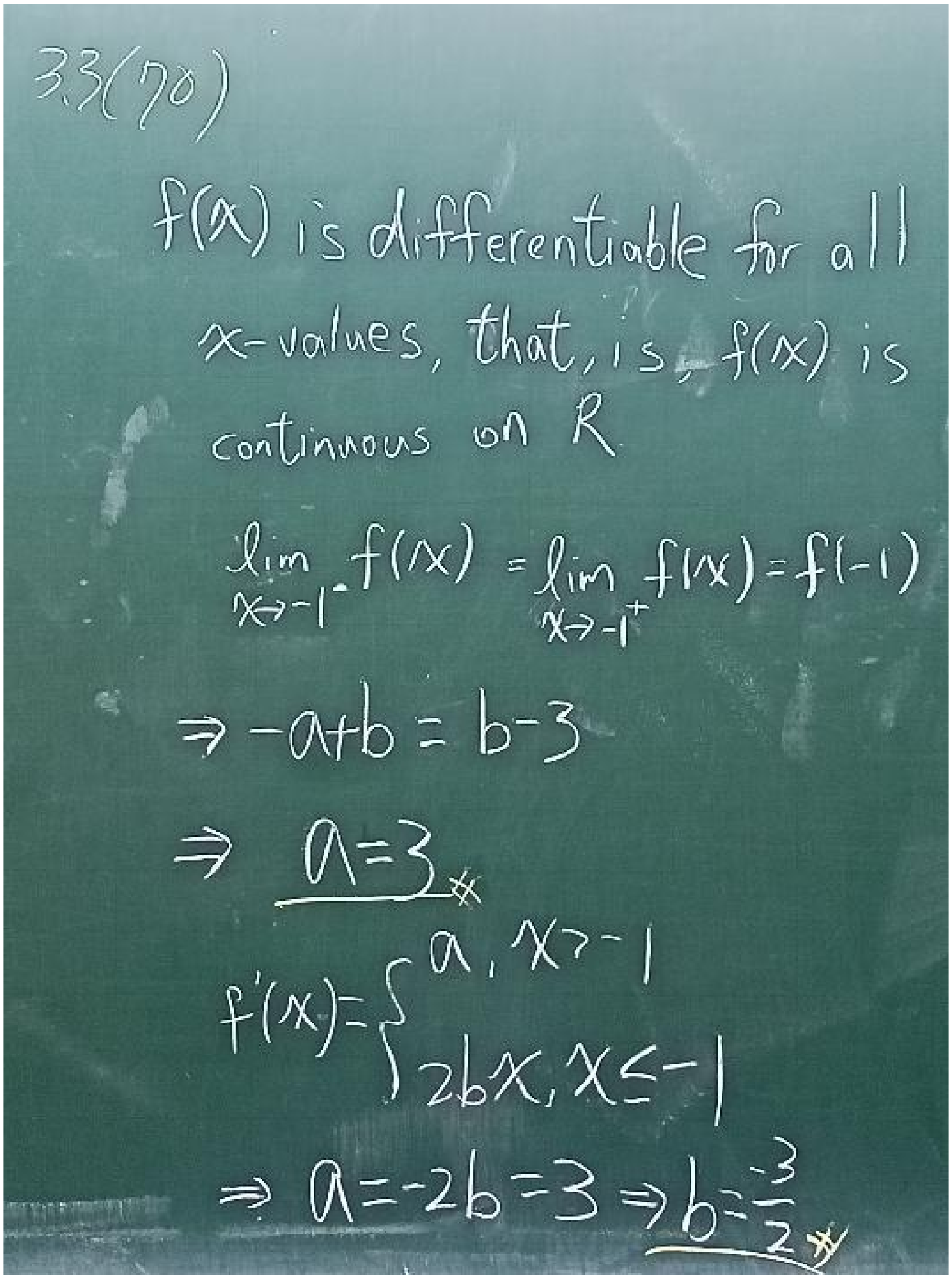
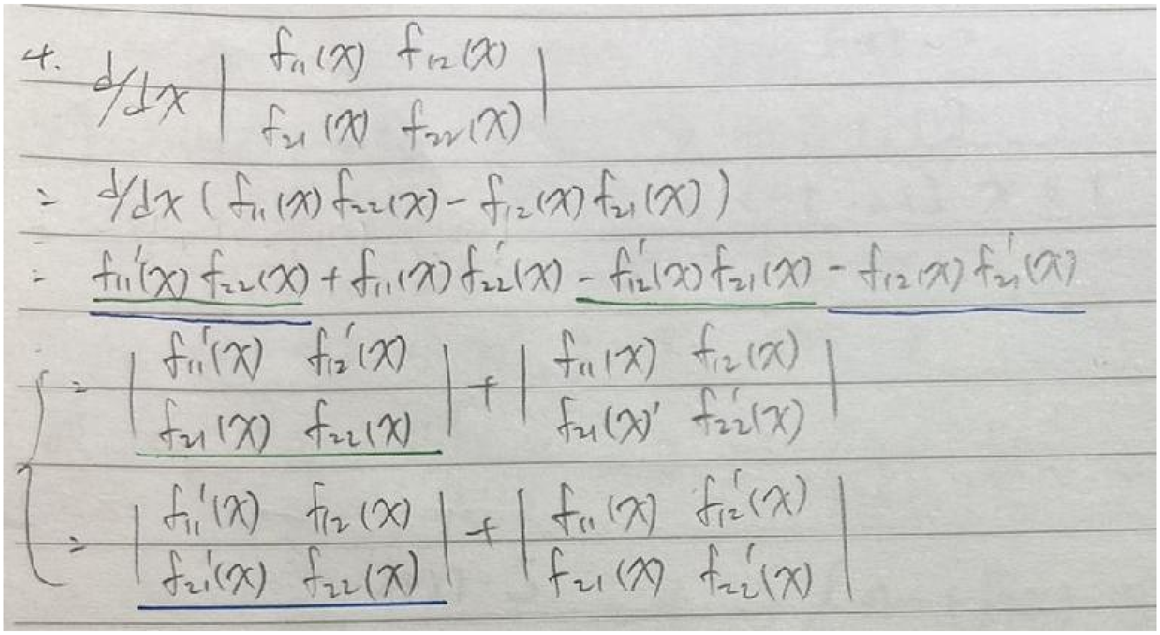


Figure 9: Solution to Section 3.3, problem 70



$$4.\quad \frac{d}{dx}\begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix}$$

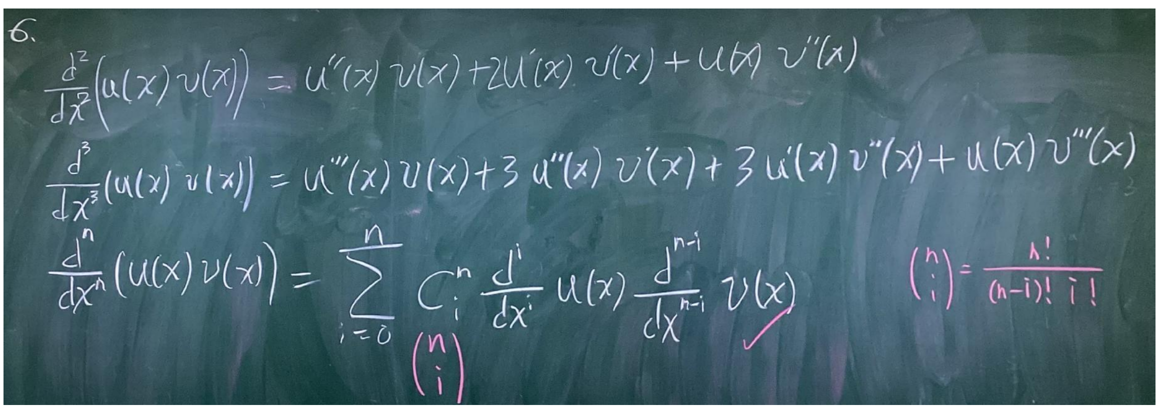
$$= \frac{d}{dx}\left(f_{11}(x) f_{22}(x) - f_{12}(x) f_{21}(x) \right)$$

$$= f_{11}'(x) f_{22}(x) + f_{11}(x) f_{22}'(x) - f_{12}'(x) f_{21}(x) - f_{12}(x) f_{21}'(x)$$

$$= \begin{vmatrix} f_{11}'(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x)' & f_{22}'(x) \end{vmatrix}$$

$$= \begin{vmatrix} f_{11}'(x) & f_{12}(x) \\ f_{21}'(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}'(x) \end{vmatrix}$$

Figure 10: Solution to Homework 03, problem 5



6.

$$\frac{d^2}{dx^2}\left(u(x) v(x)\right) = u''(x) v(x) + 2u'(x) v'(x) + u(x) v''(x)$$

$$\frac{d^3}{dx^3}\left(u(x) v(x)\right) = u'''(x) v(x) + 3 u''(x) v'(x) + 3 u'(x) v''(x) + u(x) v'''(x)$$

$$\frac{d^n}{dx^n}\left(u(x) v(x)\right) = \sum_{i=0}^{n} C^n_i \frac{d^i}{dx^i} u(x) \frac{d^{n-i}}{dx^{n-i}} v(x)$$

$$\binom{n}{i} \qquad \binom{n}{i} = \frac{n!}{(n-i)!\, i!}$$

Figure 11: Solution to Homework 03, problem 6