Brief solutions to selected problems in homework 01

1. Section 2.2: Solutions, common mistakes and corrections:

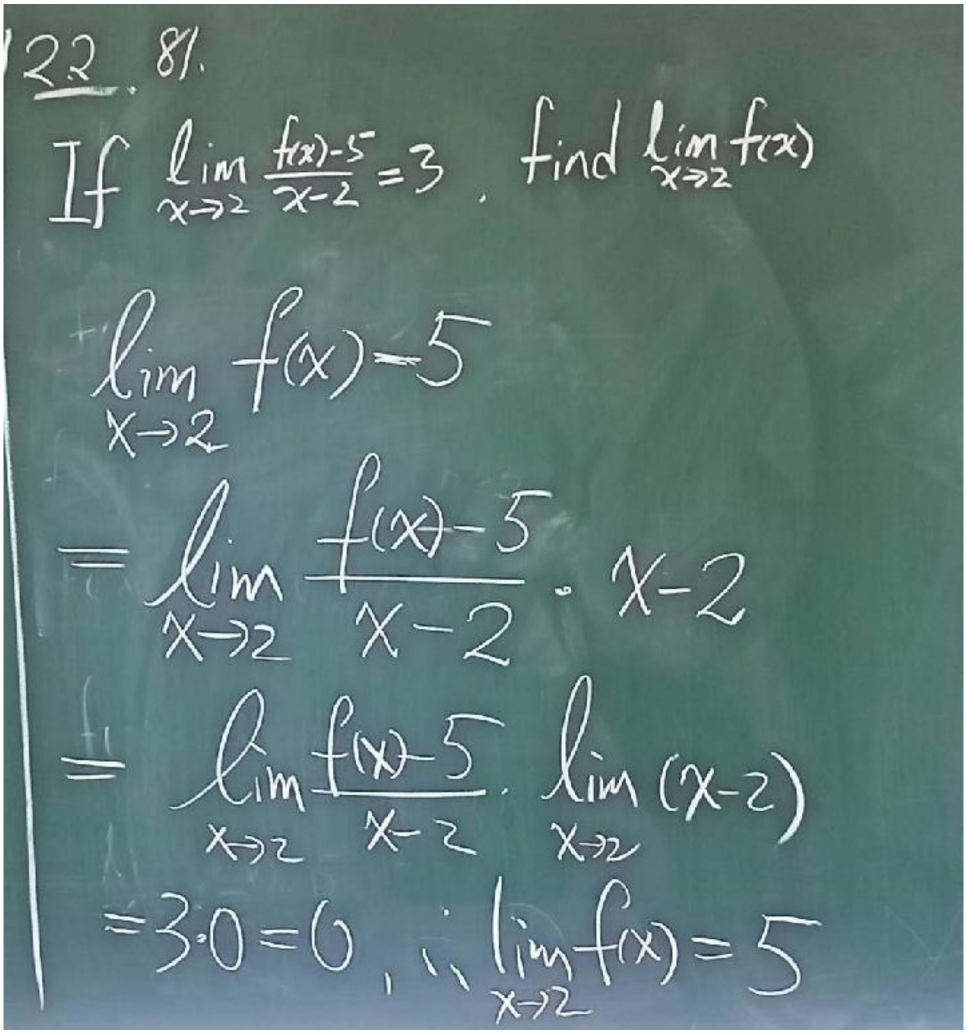


Figure 1: Section 2.2, problem 81

Remark: The idea: Since "the limit of the ratio" is nonzero and "the limit of the denominator" is zero, it follows that "the limit of the numerator" must be zero, otherwise the ratio will diverge. The idea can be supported by the Limit Laws as detailed above.

2. Problem 2:

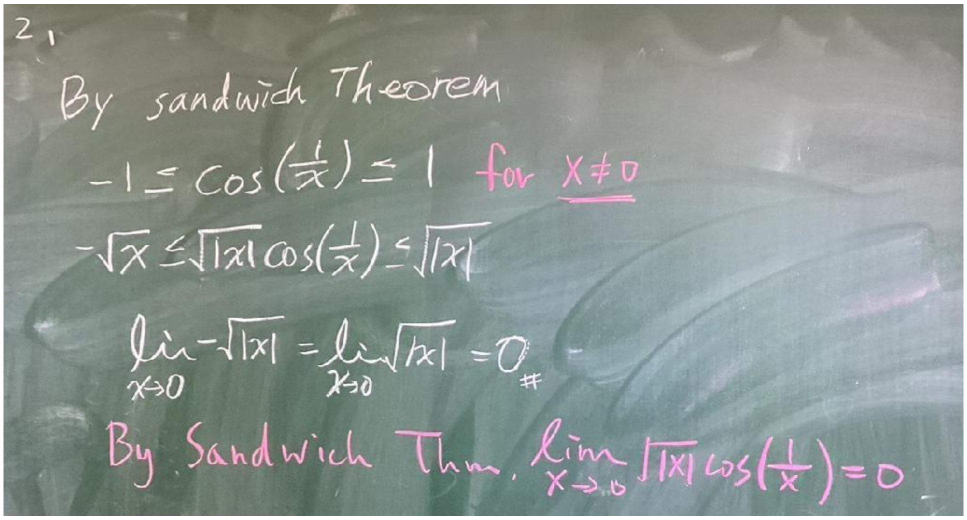


Figure 2: Homework 01, problem 2, a minor mistake

3. Section 2.3: Solutions, common mistakes and corrections:

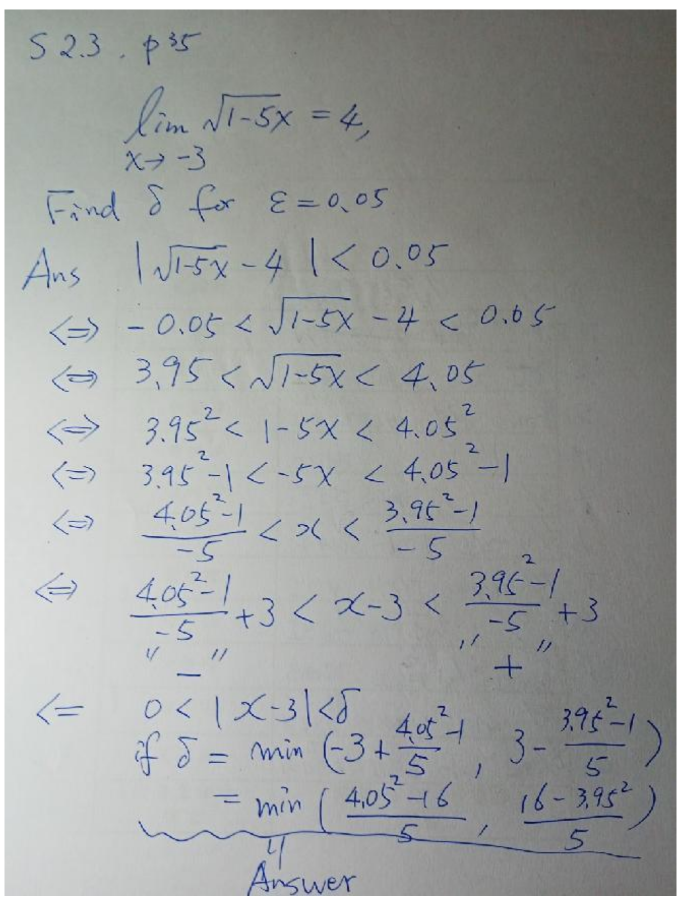


Figure 3: Section 2.3, problem 35

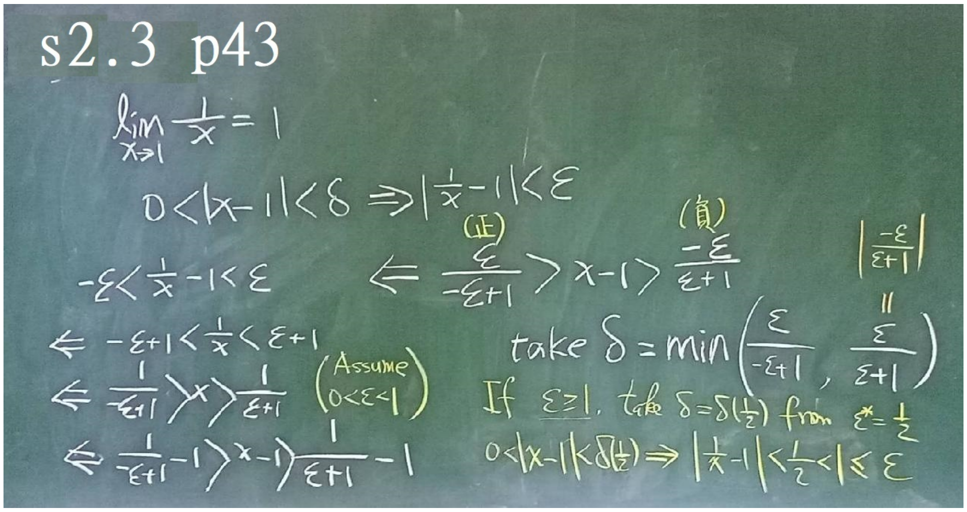


Figure 4: Section 2.3, problem 43.

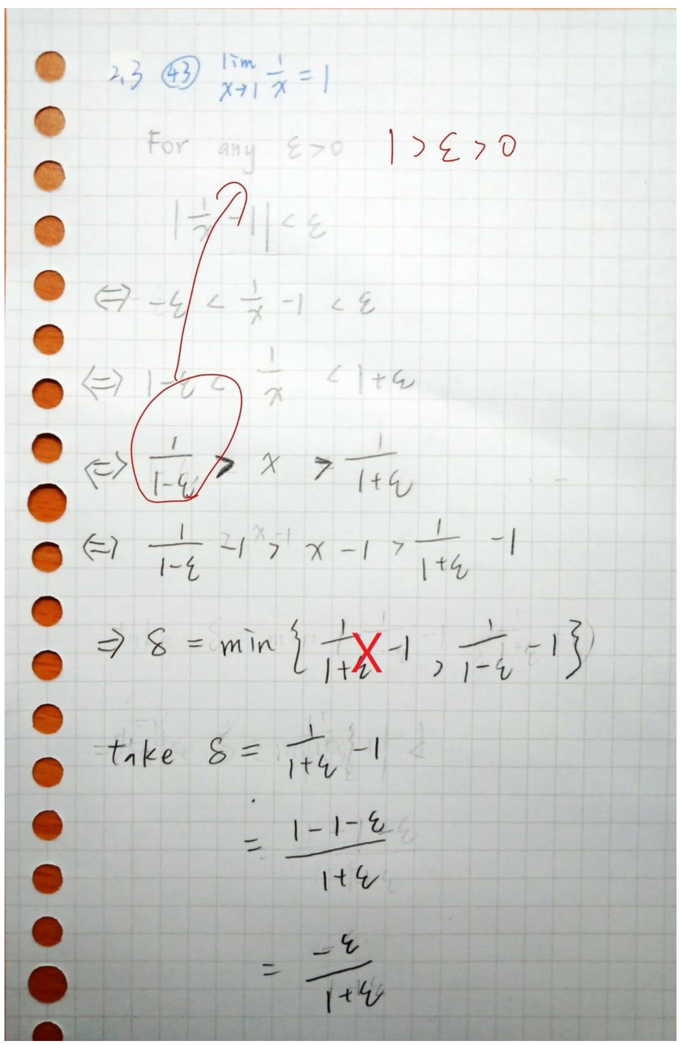


Figure 5: Section 2.3, problem 43: mistake 1

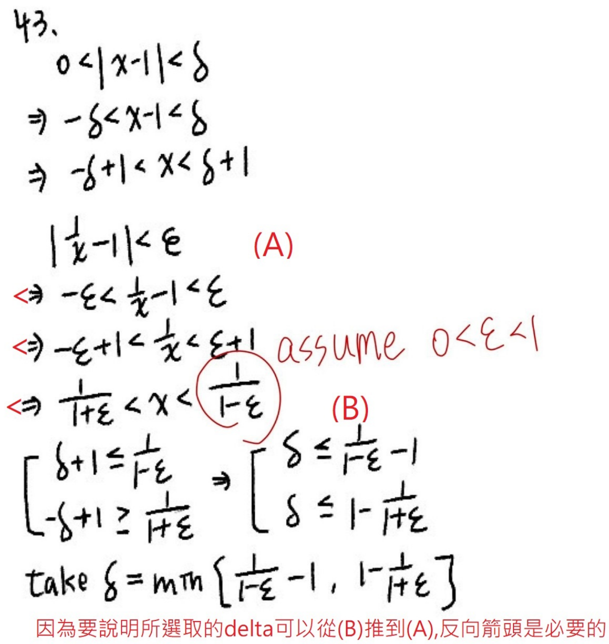


Figure 6: Section 2.3, problem 43: mistake 2

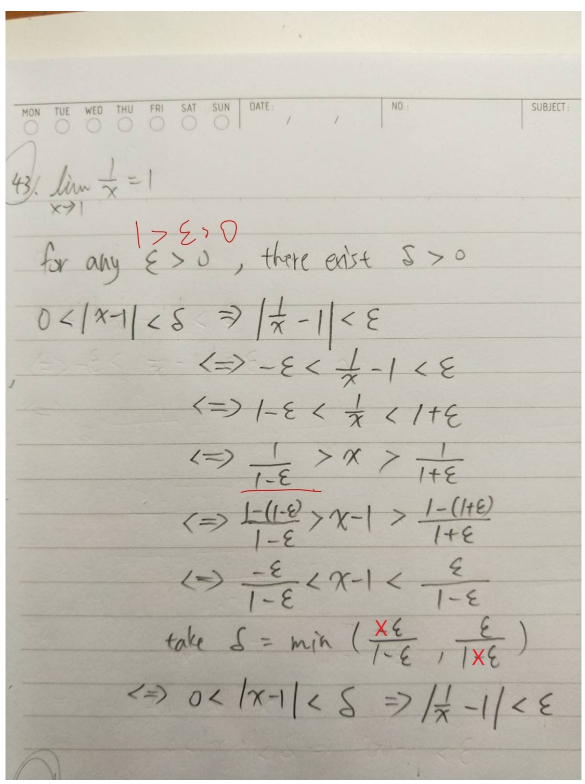


Figure 7: Section 2.3, problem 43: mistake 3

4. Problem 4: False. Counter example:

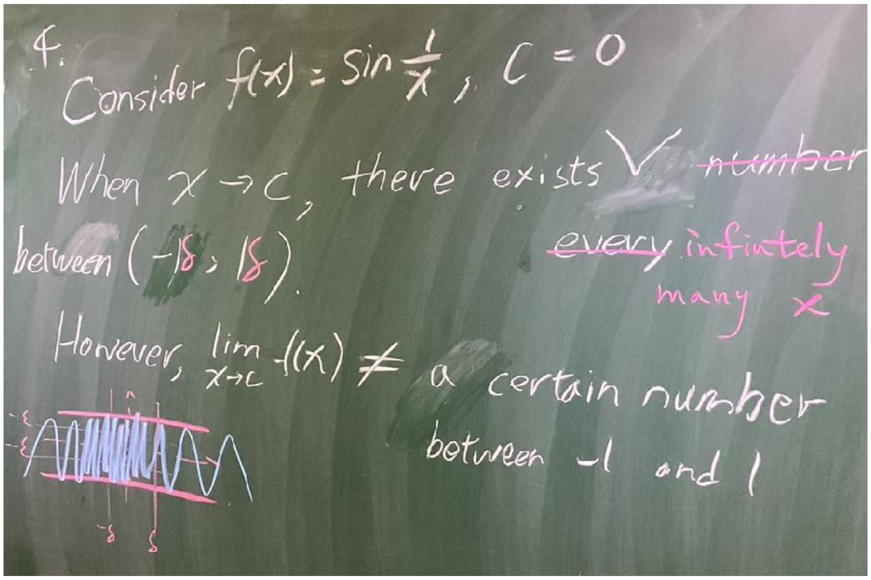
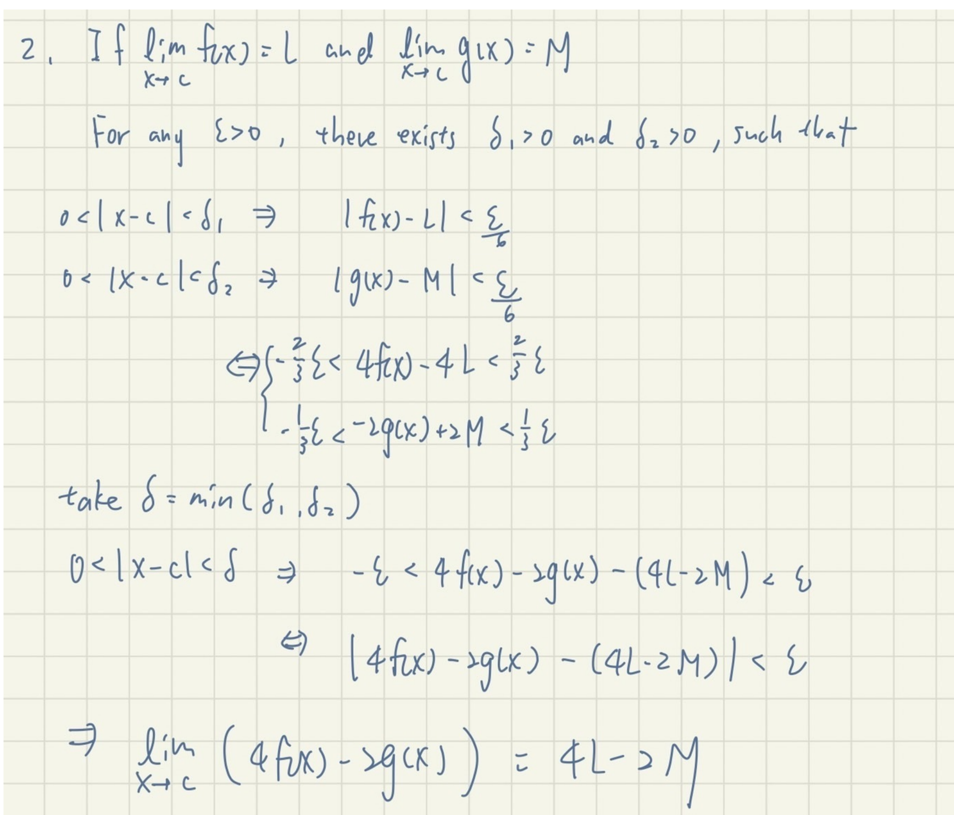


Figure 8: Homework 01, problem 4, a counter example

5. Problem 5:


2. If $\lim_{x\to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$

For any $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$0 < |x-c| < \delta_1 \Rightarrow |f(x) - L| < \frac{\varepsilon}{6}$

$0 < |x-c| < \delta_2 \Rightarrow |g(x) - M| < \frac{\varepsilon}{6}$

$\Leftrightarrow \begin{cases} -\frac{2}{3}\varepsilon < 4f(x) - 4L < \frac{2}{3}\varepsilon \\ -\frac{1}{3}\varepsilon < -2g(x) + 2M < \frac{1}{3}\varepsilon \end{cases}$

take $\delta = \min(\delta_1, \delta_2)$

$0 < |x-c| < \delta \Rightarrow -\varepsilon < 4f(x) - 2g(x) - (4L - 2M) < \varepsilon$

$\Leftrightarrow |4f(x) - 2g(x) - (4L-2M)| < \varepsilon$

$\Rightarrow \lim_{x \to c} (4f(x) - 2g(x)) = 4L - 2M$

Figure 9: Homework 01, problem 5