Hyperbolic functions (SKIP inverse hyperbolie fins) Sinh  $x = \frac{e^{x} - e^{x}}{2}$  (hyperbolic)  $\cosh x = \frac{e^{x} + e^{-x}}{2}$  (cosine) tanhx = Sinhx/coshx cothx = coshx/sinhx (x =0) Sechx = 1/coshx (Schx = 1/Sinhx (x+a)

Sinh ash (i) a // b tanh Coth
Sech Csch b-a=1 (ii) Diagnal entries are reciprocal to each other (111) Any corner entry is the product of two neighboring Corner entries

Other algebraic identities

Sinhzx = 
$$\frac{e^{x} - e^{-2x}}{e^{x}}$$

=  $\frac{(e^{x} + e^{x})(e^{x} - e^{x})}{e^{x}} = 2\sin kx$  coshx

 $\cos k 2x = \frac{e^{x} + e^{-2x}}{e^{x}}$ 

=  $\frac{1}{2}((e^{x} - e^{x})^{2} - 2) = 2\cosh x - 1$ 

=  $\frac{1}{2}((e^{x} - e^{x})^{2} + 2) = 2\sin kx + 1$ 

Sinh  $(x \pm y) = \sinh x$  coshy  $\pm \cosh x$  sinhy cosh  $(x \pm y) = \cosh x$  coshy  $\pm \sinh y$  sinhy

Kennark  $U'' + k^2 u = 0 \Rightarrow u(x) = Sinkx$  $\begin{cases} U' + k^{2}u = 0 \\ U(0) = 0 (1) \Rightarrow U(x) = \frac{1}{k} sinkx \\ U(0) = 1 (0) \qquad (coskx) \end{cases}$  $\Rightarrow U(x) = \stackrel{\pm}{e}^{kx}$ (or Sinhx, coshx) U-ku=0  $\begin{cases} u' - k' u = 0 \\ u(0) = 0 \\ u(0) = 1 \\ 0 \end{cases}$  $\Rightarrow U(X) = \frac{1}{k} Sihhkx$ 

(coshkx)

Derivative of hyporbolic fors

$$\frac{d}{dx} Sinhx = \frac{d}{dx} \left( \frac{e^{x} - e^{-x}}{2} \right) = \frac{e^{x} + e^{-x}}{2} = coshx$$

$$\frac{d}{dx} coshx = \frac{d}{dx} \left( \frac{e^{x} + e^{-x}}{2} \right) = \frac{e^{x} - e^{-x}}{2} = Sinhx$$

$$\frac{d}{dx} tanhx = \frac{d}{dx} \left( \frac{Sinhx}{coshx} \right) = \frac{cosh^{2}x - sinhx}{cosh^{2}x} = sechx$$

$$\frac{d}{dx} cothx = \frac{d}{dx} \left( \frac{coshx}{sinhx} \right) = \frac{sinhx - coshx}{sinhx} = -(sch^{2}x)$$

$$\frac{d}{dx} sechx = \frac{d}{dx} \left( coshx \right) = \frac{-sinhx}{cosh^{2}x} = -tanhx sechx$$

$$\frac{d}{dx} cschx = \frac{d}{dx} \left( sinhx \right) = \frac{-coshx}{sinh^{2}x} = -cothx cschx$$

$$\frac{d}{dx} cschx = \frac{d}{dx} \left( sinhx \right) = \frac{-coshx}{sinh^{2}x} = -cothx cschx$$

Eg 2 ( 
$$\frac{1}{3x} \frac{1}{5anh} \frac{1}{51+x^2}$$
).  $\frac{x}{51+x^2}$ 

Eg 2 (  $\frac{1}{5anhx} \frac{1}{3x}$ 

=  $\frac{1}{5anhx} \frac{1}{5anhx} \frac{1}{3x}$ 

=  $\frac{1}{5anhx} \frac{1}{5anhx} \frac{1}{5anhx}$ 

Eg3 
$$\int \sinh x \, dx$$
  
 $\int \sinh x = (e^{x} - e^{x})^{2}$   
 $= e^{x} - 2 + e^{x}$   
 $= \frac{1}{2}((e^{x} + e^{x}) - 1)$   
 $= \frac{1}{2}(\cosh 2x - 1) \cdot dx$   
 $= \frac{1}{2}(f \sinh 2x - x) + C$   
 $= \frac{1}{2}(\sinh 2x - x) + C$ 

$$\begin{aligned}
&= \begin{cases} 4 & \begin{cases} \ln 2 & x \\ 4 & e \leq 1 \\ 4 & e \leq 2 \end{cases} \\
&= \begin{cases} -2 & e \leq -1 \\ 2 & e \leq -1 \end{cases} \\
&= 2 & \begin{cases} 2x & 2x \\ 2 & e \leq -1 \end{cases} \\
&= 2 & \begin{cases} 2x & 2x \\ 2 & e \leq -1 \end{cases} \\
&= 2 & \begin{cases} 2x & 2x \\ 2 & e \leq -1 \end{cases} \\
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