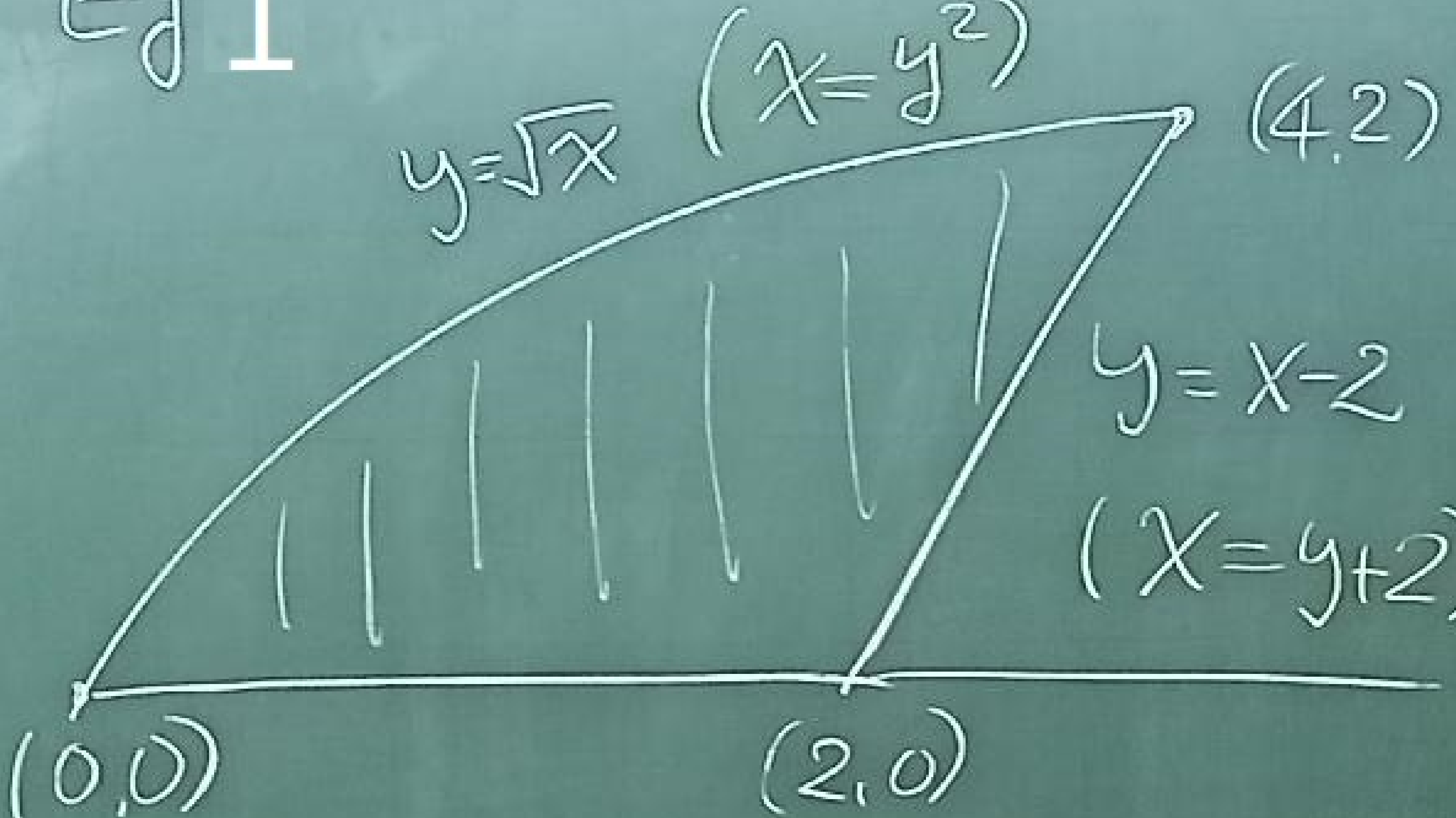
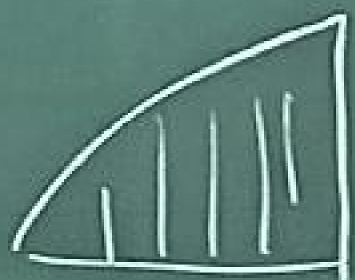
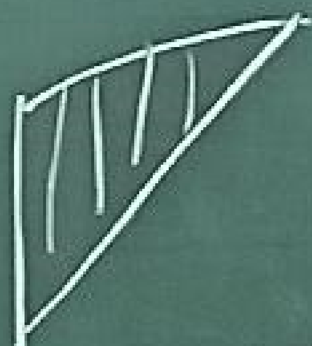
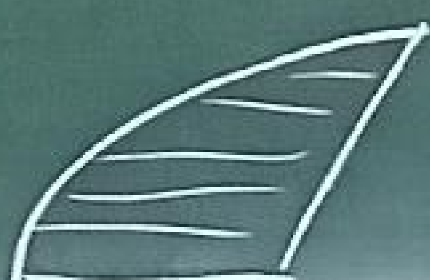
Area between Curves

Eg 1



$A =$  $+$  $(\int dx)$ (I)

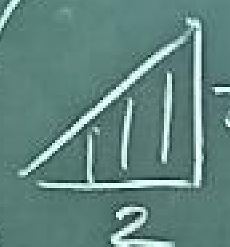
$=$  $-$  $(\int dx)$ (II)

$=$  $(\int dy)$ (III)

Sol.

$$(\mathrm{I}) = \int_0^2 (\underset{上}{\sqrt{x}} - \underset{下}{0})\,dx + \int_2^4 (\underset{上}{\sqrt{x}} - \underset{下}{(x-2)})\,dx$$

$$(\mathrm{II}) = \int_0^4 (\sqrt{x} - 0)\,dx - \int_2^4 ((x-2) - 0)\,dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 - \frac{1}{2} 2 \cdot 2 \quad ($$  $$)$$

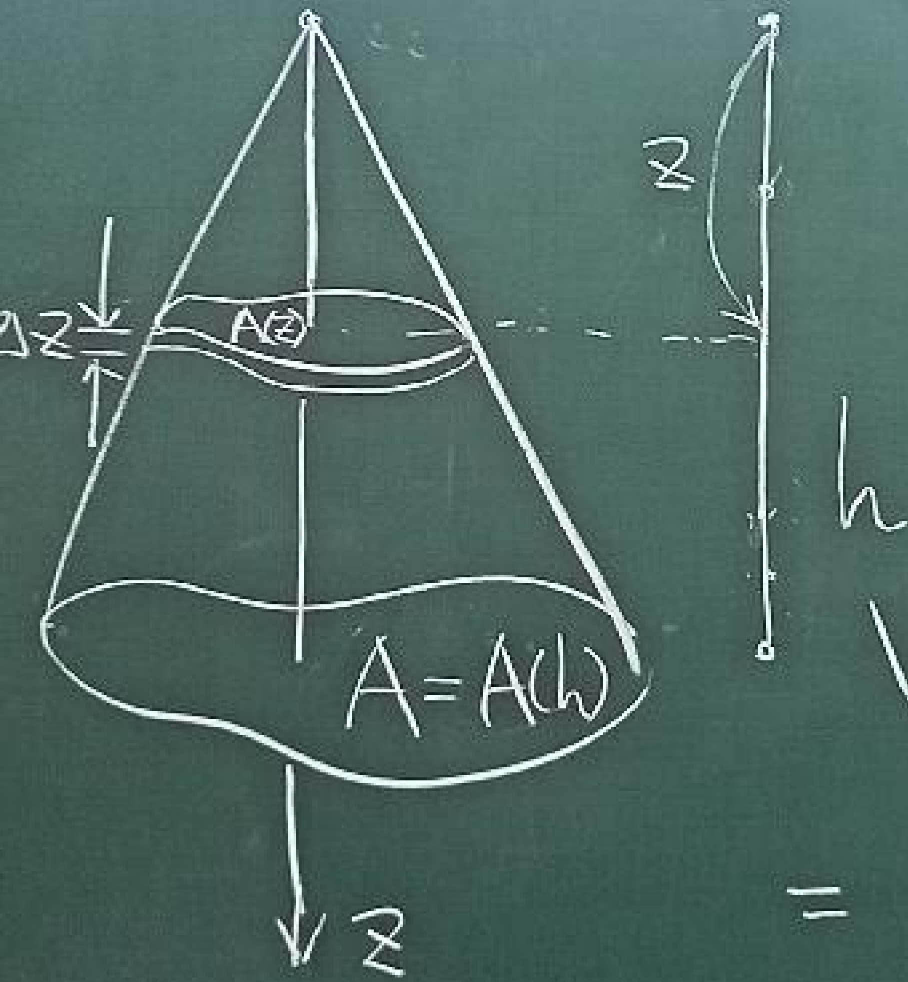
$$= \frac{16}{3} - 2 = \frac{10}{3}$$

$$(\mathrm{III}): \int_{y=0}^{2} \underset{右}{(y+2)} - \underset{左}{y^2}\,dy$$

$$= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_0^2 = \frac{10}{3}$$

Volume by Cross Section

Eg 1



$$\frac{A(z)}{A(h)} = \left(\frac{z}{h}\right)^2$$

$$A(z) = \frac{A}{h^2} z^2$$

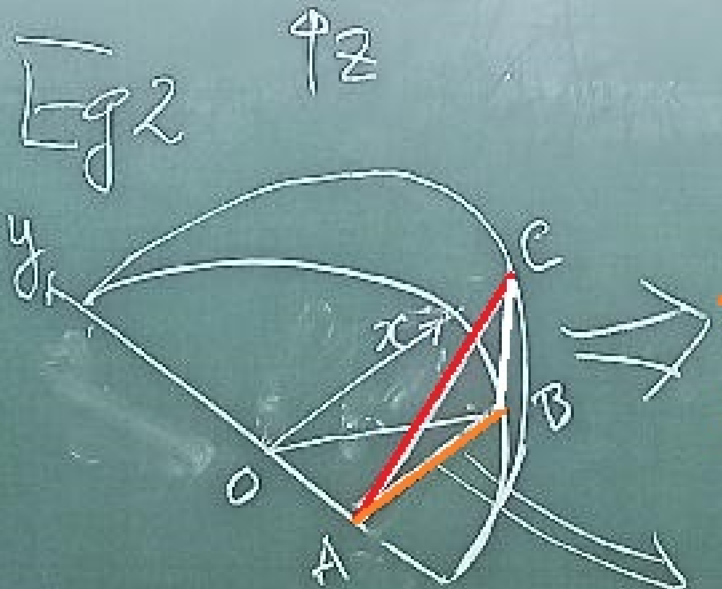
$$\Delta V(z) \cong A(z) \Delta z$$

$$V = \lim_{n \to \infty} \sum_{k=1}^{n} A(z_k) \Delta z_k$$

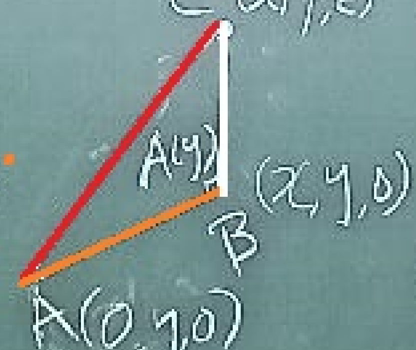
$$= \int_0^h A(z)\, dz = \int_0^h \frac{A}{h^2} z^2\, dz$$

$$= \frac{1}{3} A h$$

Eg2

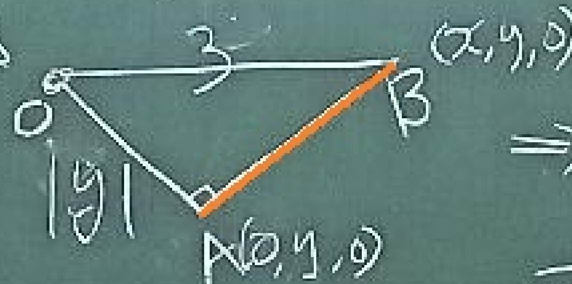


Method I: cut along "y = constant"



$$V=\int_{-3}^{3} A(y)\,dy$$

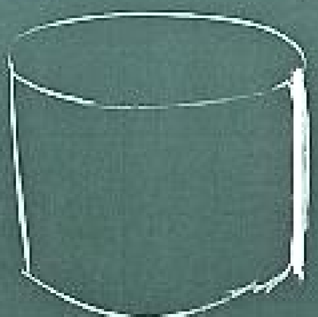
$$A(y)=\frac{1}{2}\,\overline{AB}\cdot\overline{BC}$$



$$\Rightarrow \overline{AB}=\sqrt{3^2-y^2}$$

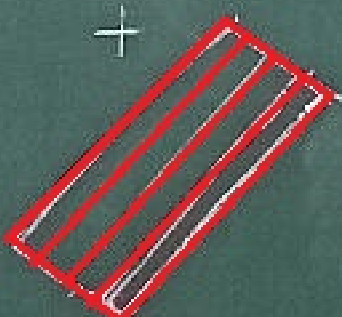
$$\overline{BC}=z=x=\overline{AB}$$

$$\therefore\ \overline{BC}=\overline{AB}=\sqrt{9-y^2}$$



$x^2+y^2=9$

+



$x=z$

+

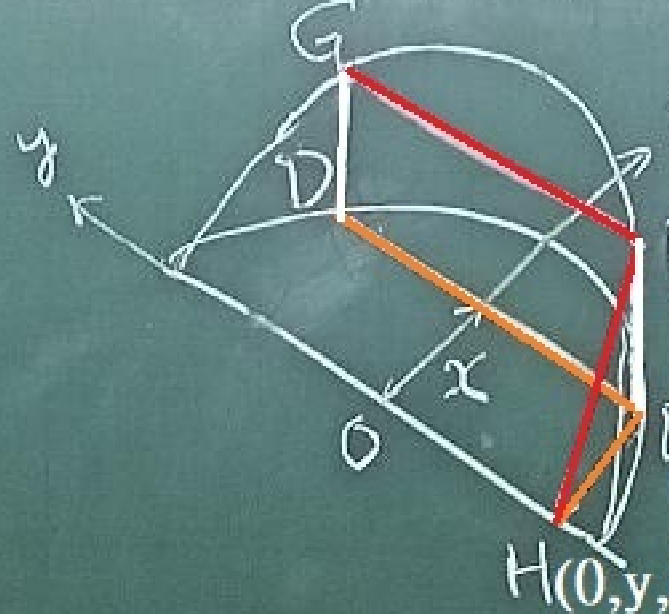
$x=0$

$$A(y)=\frac{1}{2}(9-y^2)$$

$$V=\int_{-3}^{3}\frac{1}{2}(9-y^2)=\frac{9y}{2}-\frac{y^3}{6}\Bigg|_{-3}^{3}$$

$$=18$$

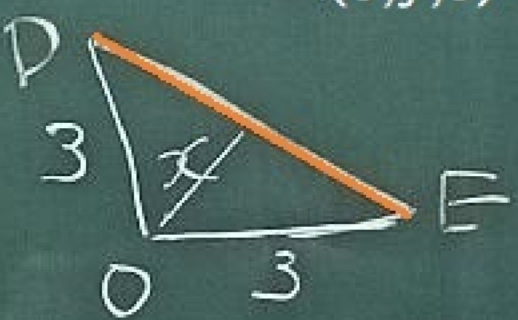
Method II, Cut along "x = const"



$$A(x) = \overline{DE}\cdot\overline{EF}$$

$$\overline{DE} = 2\sqrt{3^2-x^2}$$

$$\overline{EF} = \overline{HE} = x$$



$$V = \int_0^3 A(x)\,dx$$

$$= \int_0^3 2x\sqrt{9-x^2}\,dx$$

$$= \int_0^3 \sqrt{9-x^2}\,dx^2$$

$$= -\int_0^3 \sqrt{9-x^2}\,d(9-x^2)$$

$$= -\frac{2}{3}(9-x^2)^{\frac{3}{2}}\Big|_0^3 = 18$$