Homework 06

1. Section 3.11: problems 9, 11, 16(c,d), 17, 53, 55, 65(a,b,f(for $f(x)$ only)), 66.

2. A key point in section 3.11 is that the error of linear approximation $E(x) = f(x) - L(x)$ satisfies

$$\lim_{x \to a} \frac{f(x) - L(x)}{x - a} = 0 \tag{1}$$

or

$$f(x) - L(x) = \epsilon \cdot (x - a), \quad \lim_{x \to a} \epsilon = 0 \tag{2}$$

provided f is differentiable at $x = a$.

The following statement gives more details about the error $f(x) - L(x)$ and will be introduced in the near future. <u>Take this statement for granted for now</u>:

> If $f''(x)$ exists on $a - \delta < x < a + \delta$, then for all $x \in (a - \delta, a + \delta)$,
>
> $$f(x) - L(x) = \frac{1}{2} f''(c)(x - a)^2 \tag{3}$$
>
> for some c between x and a. The exact location of c is unknown and depends on x and a.

The formula (3) gives a more precise formula than (2) when f is twice differentiable near $x = a$.

From (3), we have an error bound

$$|f(x) - L(x)| \le \frac{1}{2} \left(\max_{c \text{ between } x \text{ and } a} |f''(c)| \right) (x - a)^2 \tag{4}$$

Use (4) to estimate the error of linear approximation (i.e. find out $|f(x) - L(x)| \le \cdots$) for problem 17 (b) of Section 3.11.

3. Section 4.1: problems 39 (need not graph it), 41, 49, 67 (absolute extreme values only), 71 (absolute extreme values only), 77.