Homework 03

1. Section 2.6: problems 29, 53, 61, 85, 92, 97.

2. Study the definitions of limits in p102, p119, p125 and p131. How would you define the four limits
$$\lim_{x\to\pm\infty} f(x) = \pm\infty?$$
Verify the statement
$$\lim_{x\to\infty} -x^3 = -\infty$$
using the definition you wrote.

3. Section 3.2: problems 17, 54, 57, 58.

4. Section 3.3: problems 5, 23, 37, 47, 55, 67, 70, 75(c). For definitions of second and higher order derivatives, see end of section 3.3.

   Hint for problem 23: Writing $f(s) = 1 + \cdots$ helps to simplify the computation.

   Hint for problem 47: Writing $r = 1 + c_1\theta^{-1} + c_2\theta^{-2} + \cdots$ helps to simplify the computation.

   Hint for problem 67: It is the definition of $f'(c)$. What is f? what is c?

   Hint for problem 70: Find a, b to make $f(x)$ continuous and differentiable at $x = -1$.

5. Use product rule to show (and memorize) that
$$\begin{aligned}\frac{d}{dx}\begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x)\end{vmatrix} &= \begin{vmatrix} f'_{11}(x) & f_{12}(x) \\ f'_{21}(x) & f_{22}(x)\end{vmatrix} + \begin{vmatrix} f_{11}(x) & f'_{12}(x) \\ f_{21}(x) & f'_{22}(x)\end{vmatrix} \\ &= \begin{vmatrix} f'_{11}(x) & f'_{12}(x) \\ f_{21}(x) & f_{22}(x)\end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f'_{21}(x) & f'_{22}(x)\end{vmatrix}\end{aligned}$$
   What are the corresponding formulae for 3 by 3 determinants, 4 by 4 determinants, etc.?

6. Apply the product rule repeatedly to get $\dfrac{d^n}{dx^n}\Big(u(x)v(x)\Big)$ in terms of derivatives of $u(x)$ and $v(x)$. Start with $n = 2$, then $n = 3, \cdots$, to find the formula.