

Study guide for quiz 08

Quiz problems include both the lecture contents and homework problems.

1. Section 14.4: Study the Chain rule for composition of differentiable functions of one or more independent variables (along with one or more intermediate variables) such as Theorem 5 (1 independent, 2 intermediate variables), Theorem 6 (1 independent, 3 intermediate variables), Theorem 7 (2 independent, 3 intermediate variables) and so on for more general cases.
2. Section 14.4: Study how to evaluate

$$\frac{d}{dx} \int_{g_1(x)}^{g_2(x)} f(x, t) dt.$$

The image shows a handwritten derivation on a chalkboard. It starts with the expression $\frac{d}{dx} \int_{g_1(x)}^{g_2(x)} f(x, t) dt = ?$. Below this, $f(x, t)$ is underlined and labeled $h(g_1(x), t)$. The next line is $\frac{d}{dx} F(g_1(x), g_2(x), g_3(x)) = ?$. This is followed by the partial derivative expansion: $= \partial_1 F \cdot \frac{dg_1}{dx} + \partial_2 F \cdot \frac{dg_2}{dx} + \partial_3 F \cdot \frac{dg_3}{dx}$. Then, a function $F(u, v, w) = \int_u^v h(w, t) dt$ is defined. An arrow points from this definition to the partial derivatives: $= \underline{F_u} \cdot g_1' + F_v \cdot g_2' + F_w \cdot g_3'$. Finally, the partial derivatives are expressed as integrals: $F_u \approx \frac{d}{du} \int_u^{c_2} h(c_3, t) dt$ and $F_v \approx \frac{d}{dv} \int_{c_1}^v h(c_3, t) dt$.

Figure 1: Guide to item 2


$$\begin{aligned}
 & \frac{d}{dv} \int_{C_1}^v h(C_3, t) dt \\
 &= h(C_3, v) \\
 &= \lim_{h \rightarrow 0} \frac{\int_{C_1}^{v+h} dt - \int_{C_1}^v dt}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\int_v^{v+h} h(C_3, t) dt}{h} = h(C_3, v)
 \end{aligned}$$


Figure 2: Guide to item 2, continued

$$\begin{aligned}
 F_\omega &\triangleq \int_{C_1}^{C_2} h(\omega, t) dt \\
 &= \int_{C_1}^{C_2} \lim_{h \rightarrow 0} \frac{h(\omega+h, t) - h(\omega, t)}{h} dt \\
 &= \int_{C_1}^{C_2} h_\omega(\omega, t) dt \\
 \text{E.g.: } \frac{d}{dx} \int_x^{x^2} \sin(x^3 - t) dt \\
 &= -\sin(x^3 - x) + \sin(x^3 - x^2) \cdot \frac{dx^2}{dx} \\
 &\quad + \int_x^{x^2} \cos(x^3 - t) \cdot 3x^2 dt
 \end{aligned}$$

Figure 3: Guide to item 2, continued

3. Section 14.5: Study the definition of directional derivative and how to compute it from definition, and alternatively how to compute it using partial derivatives when the

function is differentiable.

4. Section 14.5: Study the geometric meaning of the gradient vector. Study how to find the tangent line and normal line of a level curve of $f(x, y)$ (i.e., $\{(x, y) \mid f(x, y) = c\}$) using gradient of f .