Study guide for Final Exam

Final Exam problems include both the lecture contents and homework problems.

1. Review study guide for quiz 09-12.

2. Section 16.1, 16.2: All exam problems will be expressed in explicit mathematical symbols. So you do not need to memorize the definitions of first moments, center of mass and moments of inertia, etc. in Section 16.1 and work, circulation, flow and flux in Section 16.2 in preparing the exams.

3. Section 16.1, 16.2:

   Study the meanings of

$$\int_C f(x,y,z)\,ds,$$

$$\int_C \boldsymbol{F}(x,y,z)\cdot \boldsymbol{T}ds = \int_C \boldsymbol{F}(x,y,z)\cdot d\boldsymbol{r} = \int_C M(x,y,z)\,dx + N(x,y,z)\,dy + P(x,y,z)\,dz,$$

$$\int_C \boldsymbol{F}(x,y)\cdot \boldsymbol{n}\,ds = \oint_C \boldsymbol{F}(x,y)\cdot \boldsymbol{n}\,ds = \oint_C M(x,y)\,dy - N(x,y)\,dx$$

   and how to calculate them using a properly chosen parametrization of C: $\boldsymbol{r}(t)$, $t_0 \le t \le t_1$.

   A few points to pay attention:

   Which of them is (are) independent of the orientation of C? Which of them depend(s) on the orientation of C?

   How do you choose the parametrization $\boldsymbol{r}(t)$ so that the direction of $\boldsymbol{T}$ comply with the orientation of C?

   How is the outward normal $\boldsymbol{n}$ related to $\boldsymbol{T}$ if the parametrization of C is increasing in the counter-clockwise direction?

4. Section 16.3:

   Read "Note on Conservative Fields and Simply Connected Domains (Check_conservativeness_v01.pdf)" on the course homepage.

   Study and memorize the definitions of 'path independent', 'conservative' and 'potential function' (p984).

   Study the statement and proof of Theorem 1:'Fundamental Theorem of Line Integrals' (p985).

   Study the definition and examples (Figure 16.22) of 'connected domain' and 'simply connected domain' (p985).

   Study the relation between conservative fields and

(a) ’Gradient Fields’ (Theorem 2, p986).
(b) ’Loop Property’ (Theorem 3, p987)
(c) ’Component test’ (p988),

Which of them are equivalent to ’Conservative Fields’?

Which of them are equivalent to ’Conservative Fields’ only on simply connected domains?

Which of the implications ’$\Longleftarrow$’, ’$\Longrightarrow$’ remains valid even if the domain is not simply connected? Review homework 15, problem 4,5 for relevant examples.

For given functions $M(x,y,z), N(x,y,z), P(x,y,z)$ satisfying the component test, how does one find the potential function (if it exists) by way of direct integration? See Example 3, page 98 for details.

5. Section 16.3:

   Skip ”Exact Differential Forms” on page 991-992.