Brief solutions to selected problems in homework 14

1. Section 15.7: Solutions, common mistakes and corrections:

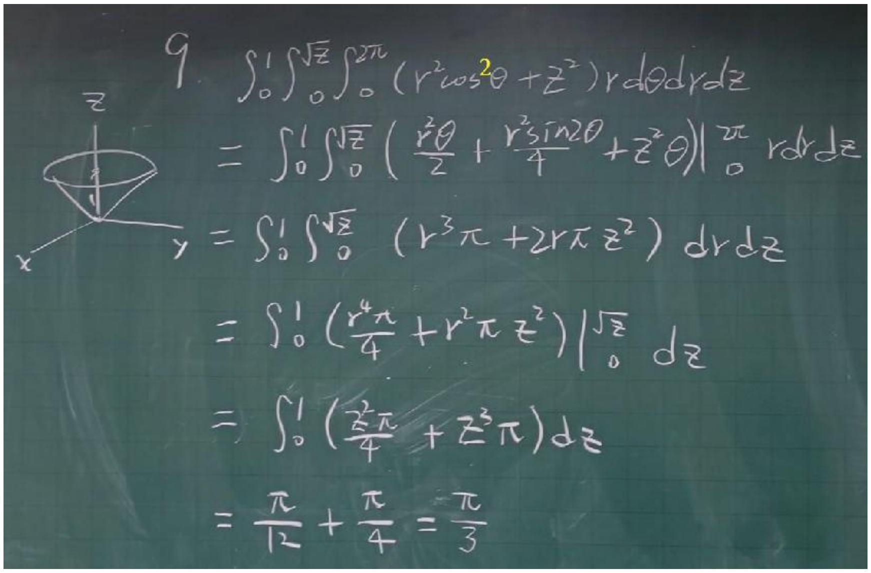

Figure 1: Solution to Section 15.7, problem 9

Remark: It is worth remembering that $\int_0^{2\pi} \cos^2\theta\, d\theta = \int_0^{2\pi} \sin^2\theta\, d\theta = \int_0^{2\pi} \frac{1}{2} d\theta$ as we mentioned in class.

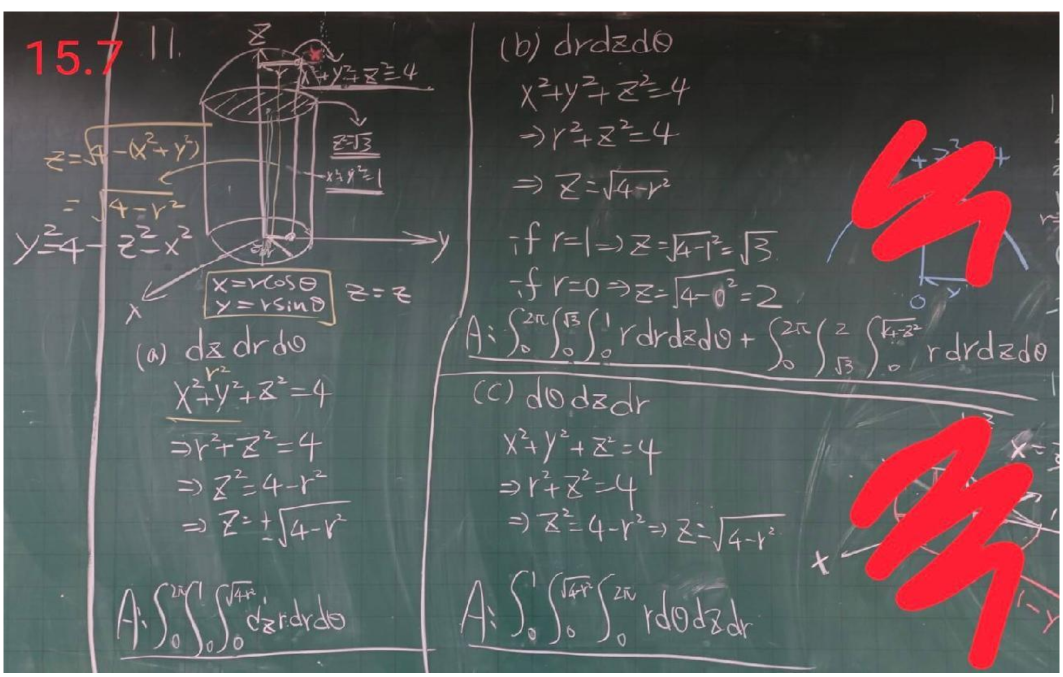

Figure 2: Solution to Section 15.7, problem 11

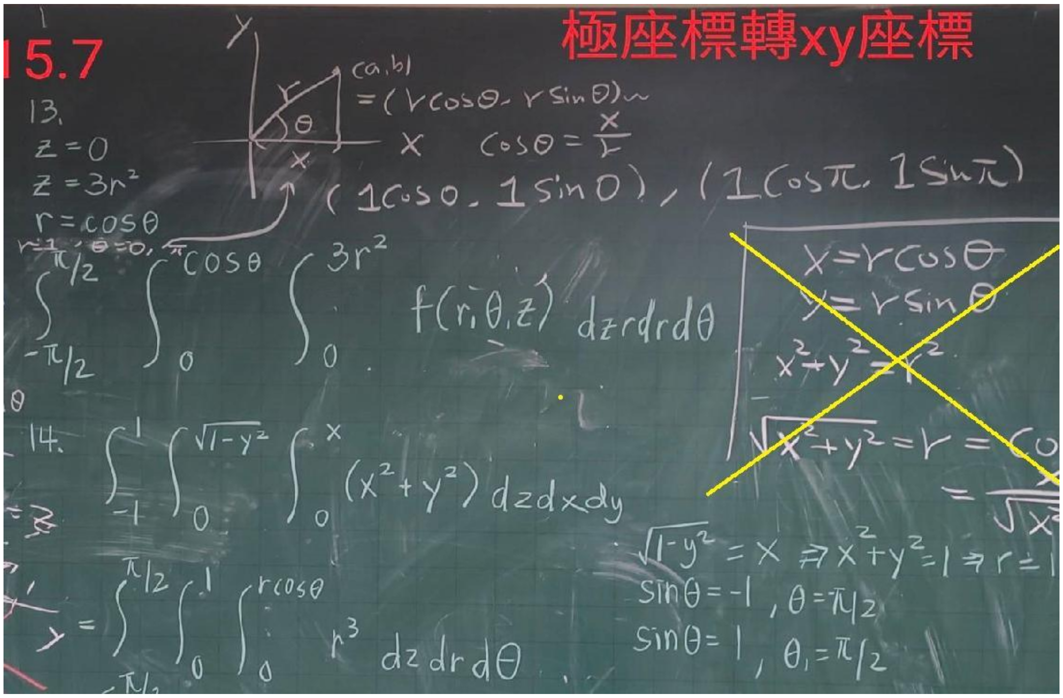

Figure 3: Solution to Section 15.7, problem 13-14

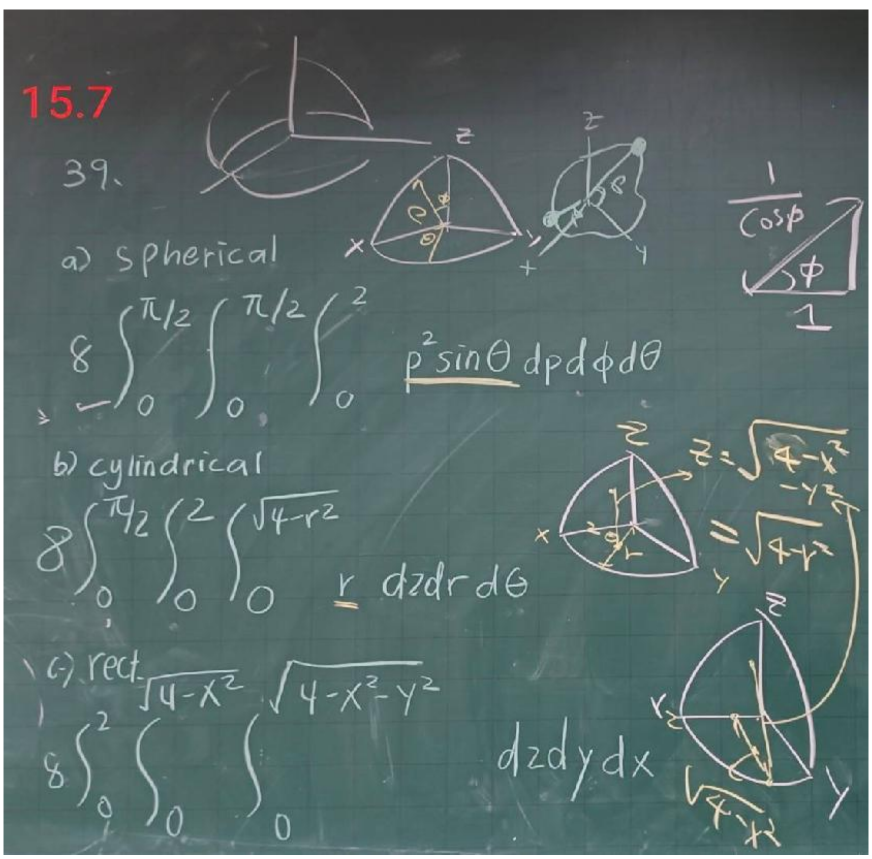

Figure 4: Solution to Section 15.7, problem 39

31

31. Let D be the region in Exercise 11. Set up the triple integrals in spherical coordinates that give the volume of D using the following orders of integration.

a. $d\rho\, d\phi\, d\theta$ **b.** $d\phi\, d\rho\, d\theta$

11. Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$. Set up the triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration.

a. $dz\, dr\, d\theta$ **b.** $dr\, dz\, d\theta$ **c.** $d\theta\, dz\, dr$

a. $\boxed{x^2+y^2=1}$

$$\Rightarrow \rho^2 \sin^2\phi = 1$$
$$\rho \sin\phi = 1$$
$$\rho = \csc\phi$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_0^{2} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta + \int_0^{2\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_0^{\csc\phi} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

b. $$\int_0^{2\pi}\int_1^{2}\int_{\cancel{0}\ \frac{\pi}{6}}^{\sin^{-1}(\frac{1}{\rho})} \rho^2 \sin\phi \, d\phi\, d\rho\, d\theta + \int_0^{2\pi}\int_0^{2}\int_0^{\frac{\pi}{6}} \rho^2 \sin\phi \, d\phi\, d\rho\, d\theta + \int_0^{2\pi}\int_0^{1}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \rho^2 \sin\phi \, d\phi\, d\rho\, d\theta$$

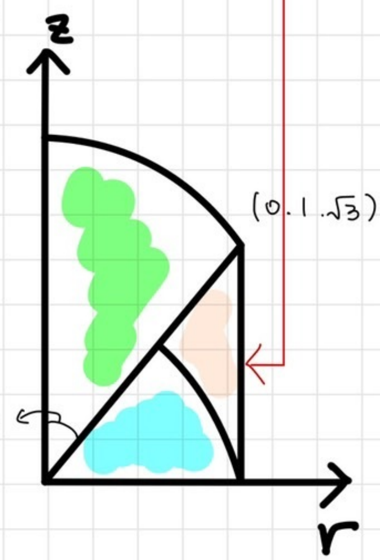

Figure 5: Solution to Section 15.7, problem 31

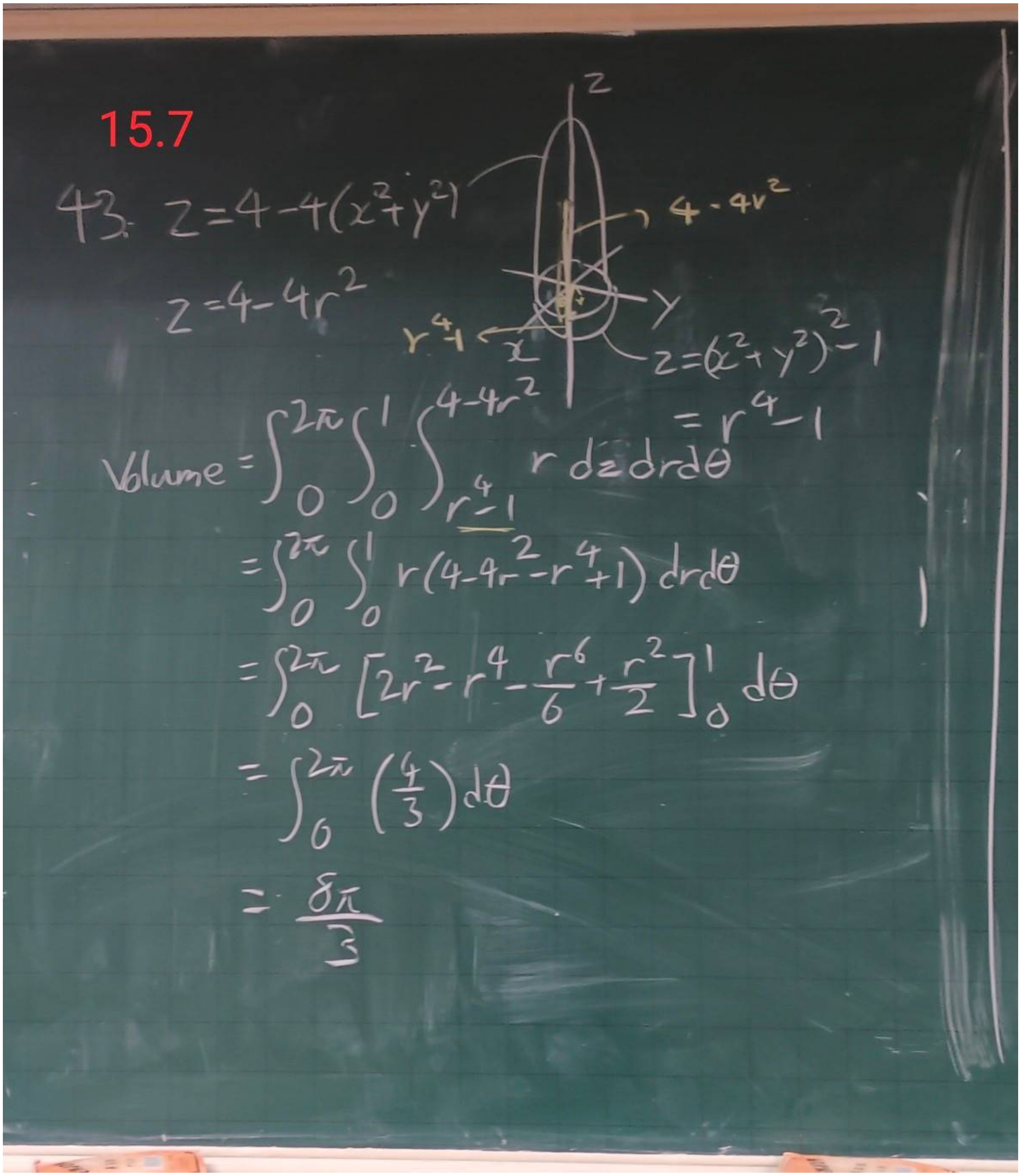

Figure 6: Solution to Section 15.7, problem 43

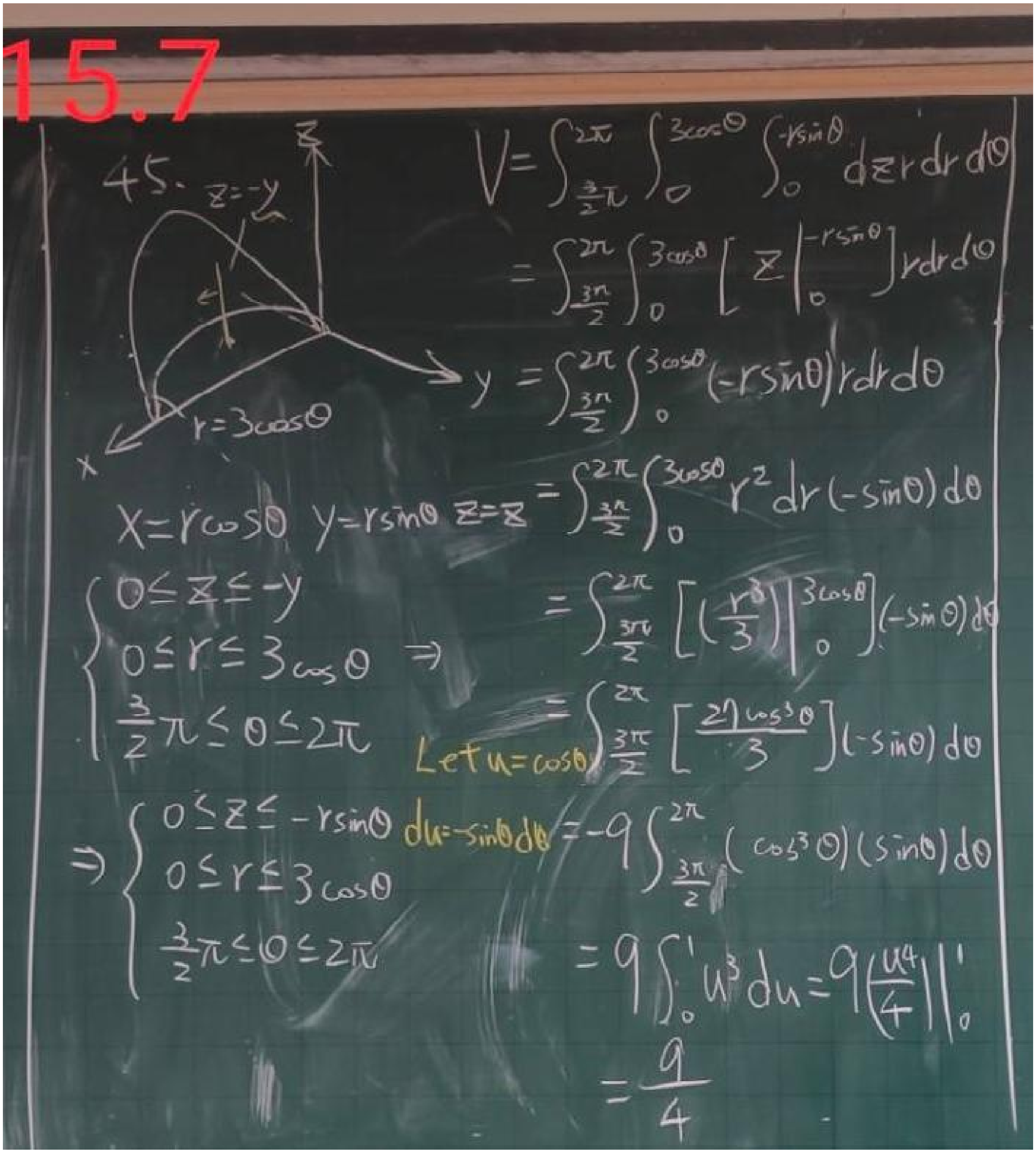

Figure 7: Solution to Section 15.7, problem 45

2. Section 15.8: Solutions, common mistakes and corrections:

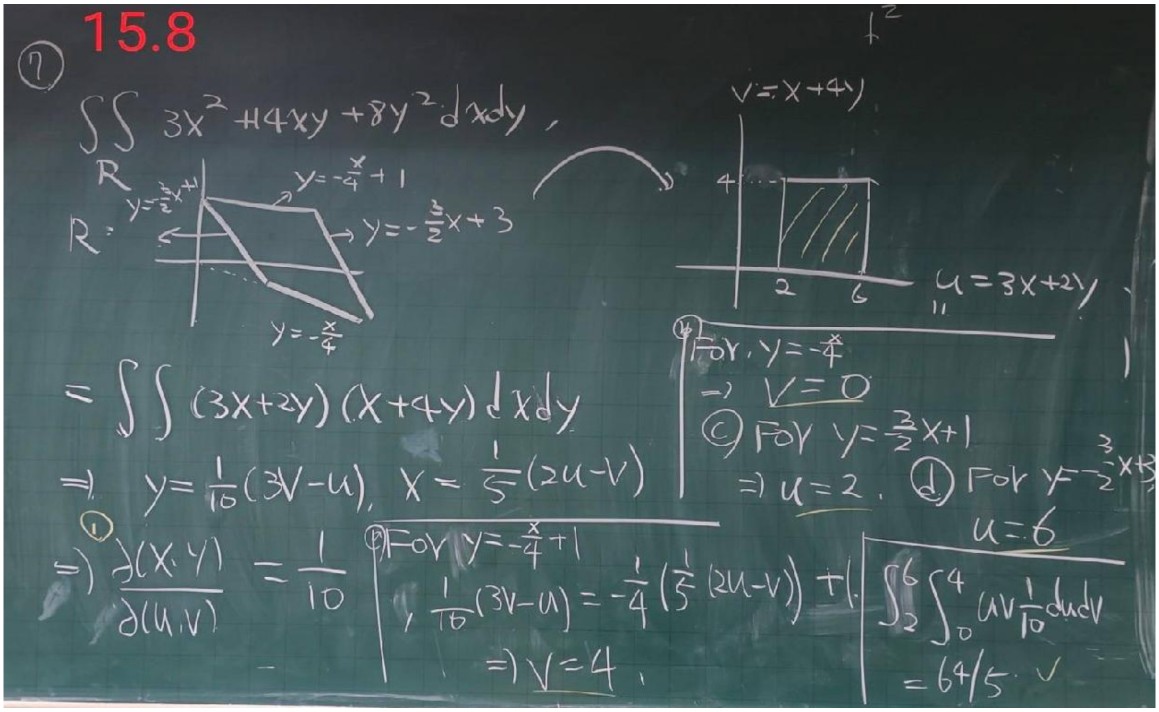

Figure 8: Solution to Section 15.8, problem 7

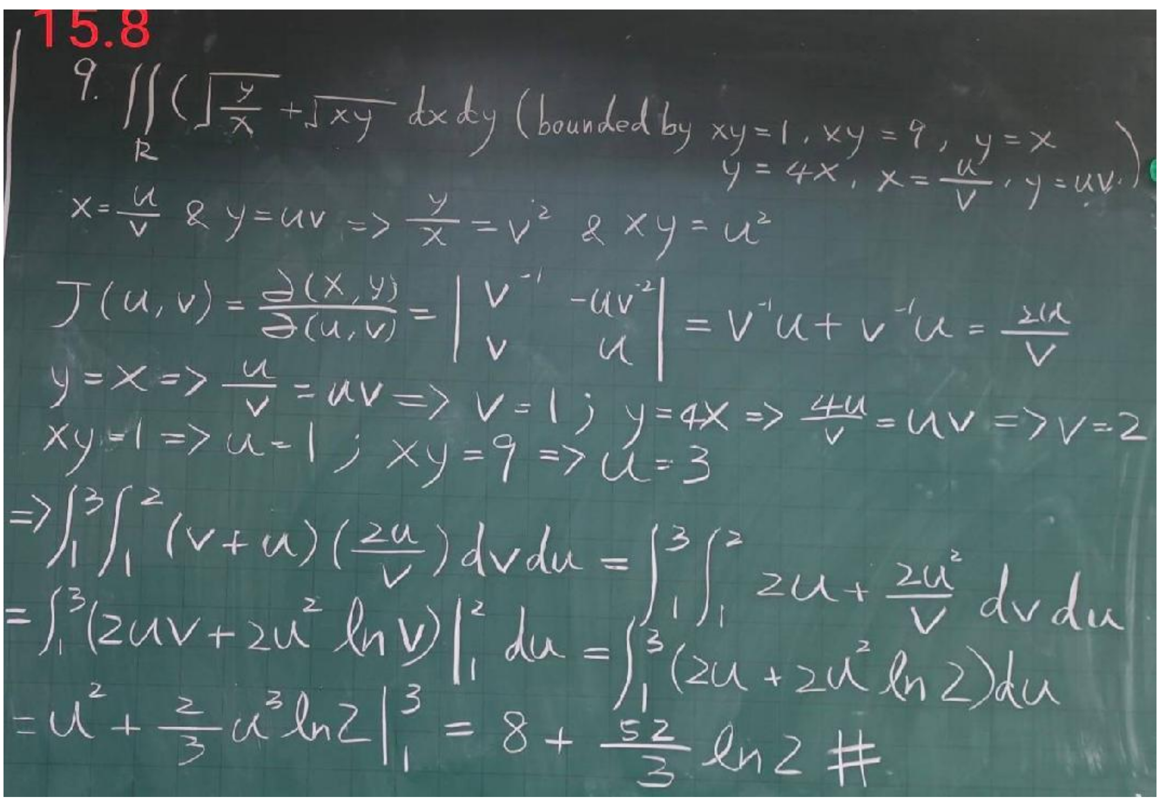

Figure 9: Solution to Section 15.8, problem 9

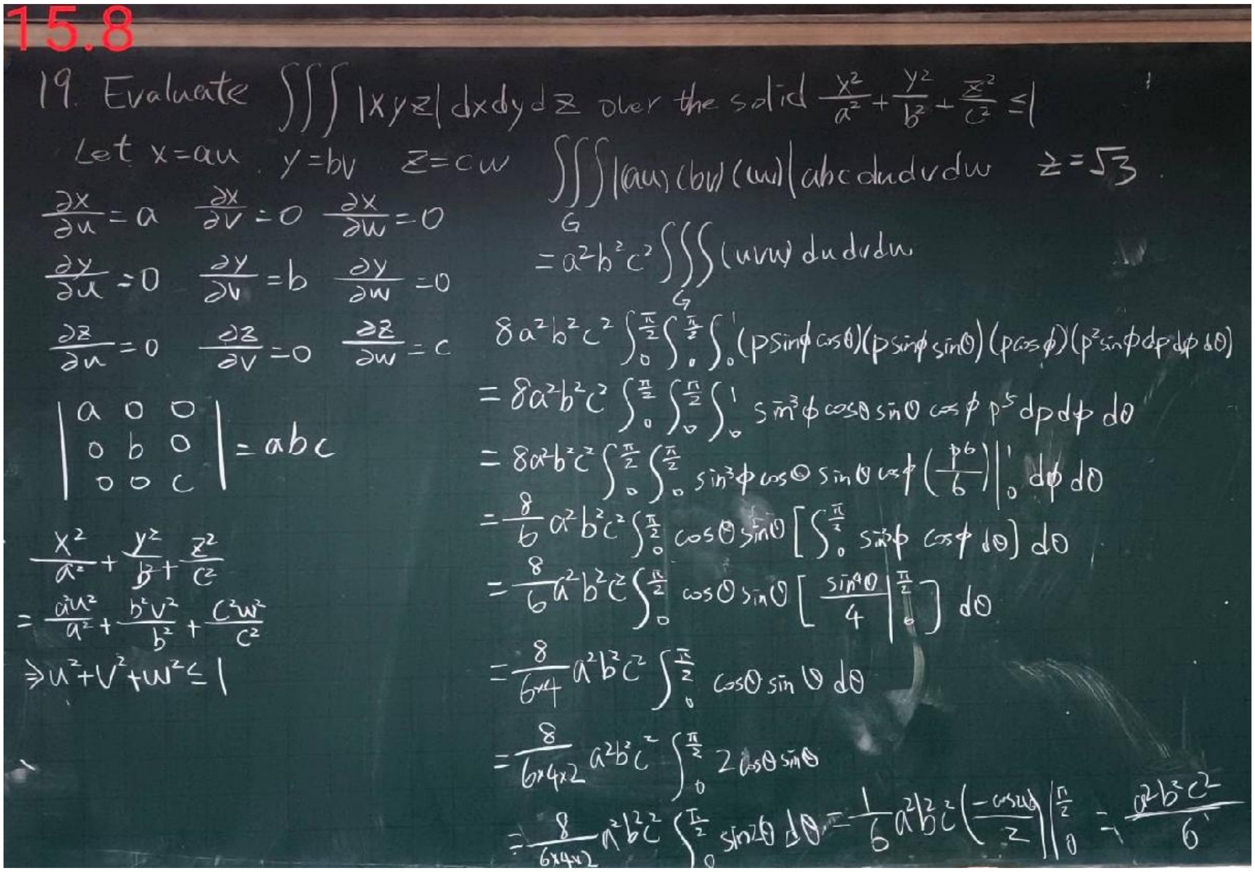

Figure 10: Solution to Section 15.8, problem 19

3. Homework 14, problem 3:

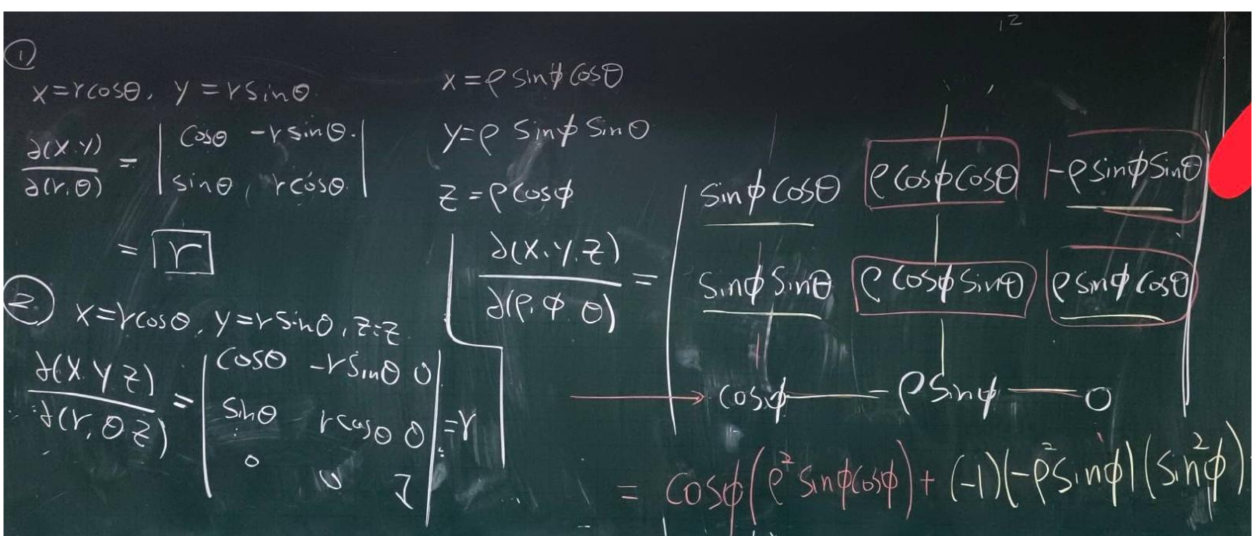

Figure 11: Solution to problem 3

4. Chap 15, Additional and Advanced Exercises:

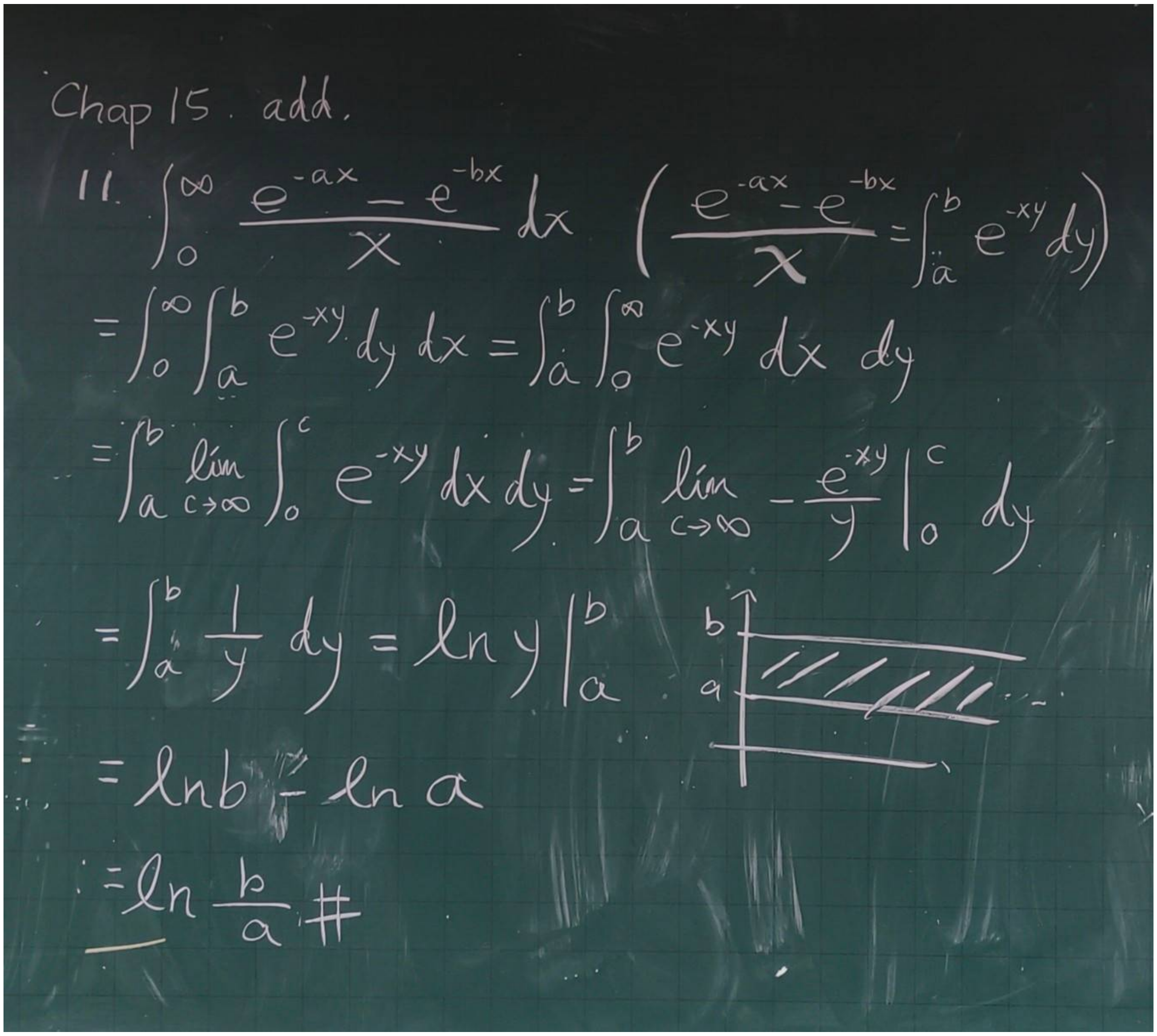

Figure 12: Solution to Chap 15, Additional and Advanced Exercises: problem 11