Calculus I, Spring 2024 (Thomas' Calculus Early Transcendentals 13ed), http://www.math.nthu.edu.tw/~wangwc/

Brief solutions to selected problems in homework 14

1. Section 15.7: Solutions, common mistakes and corrections:

= Jo JE (2 + 4 + 20) The rande = $\int_{0}^{1} \int_{0}^{\sqrt{2}} (r^{3}\pi + 2r\pi z^{2}) dr dz$ $= \int_{0}^{1} \left(\frac{r^{4}\pi}{4} + r^{2}\pi z^{2} \right) \Big|_{0}^{12} dz$ = S! (=+ 23 R) dz $=\frac{\pi}{12}+\frac{\pi}{4}=\frac{\pi}{3}$

Figure 1: Solution to Section 15.7, problem 9

Remark: It is worth remembering that $\int_0^{2\pi} \cos^2 \theta \, d\theta = \int_0^{2\pi} \sin^2 \theta \, d\theta = \int_0^{2\pi} \frac{1}{2} d\theta$ as we mentioned in class.



Figure 2: Solution to Section 15.7, problem 11

2 (ICOST. ISNT) 10.50. 1 \mathcal{O} Sin COSO 5 dzrdrdo VI- y2 dzdxdy S dzdrdQ

Figure 3: Solution to Section 15.7, problem 13-14

39. T1/2 TL/2 sin O dp 0 b) cylindrical ð dzdr d6 (-) 4-X2 14-X2dzdydx

Figure 4: Solution to Section 15.7, problem 39



Figure 5: Solution to Section 15.7, problem 31

15.7 2=4-Volume = $\int_{0}^{2\pi} \int_{0}^{1} \int_{r=1}^{4-4r^2} r dz dr d\theta$ $=\int_{0}^{2\pi}\int_{0}^{1}r(4-4r^{2}-r^{4}+1)drd\theta$ $= \int_{0}^{2\pi} \left[2r^{2} - r^{4} - \frac{r^{6}}{6} + \frac{r^{2}}{2} \right]_{0}^{1} d\Theta$ $= \int_{6}^{2\pi} \left(\frac{4}{3}\right) d\theta$

Figure 6: Solution to Section 15.7, problem 43

3000 (-Kind derdrdo) 3000 [E | -rsino]rdrdo 2= $\int_{3^{n}}^{2n} \left(\frac{3\cos^{8}}{2} \right) \mathbb{Z} \left[\frac{-r\sin^{8}}{2} \right]$ rsin0)rdrd0 30050 $\int_{\frac{3n}{2}}^{2\pi} \int_{0}^{3\omega s0} r^2 dr (-sin0) d0$ =YSTO Z=Z $= \int_{\frac{37}{2}}^{2\pi} \left[\left(\frac{r^3}{3} \right) \right]_{0}^{3\cos\theta} (-\sin\theta) d\theta$ cas O = (2x 3m 14= cosb) = 2/10530 052TC)(-sino)do 0525-rsinOdu-sinOdu $(\cos^3 0)(\sin 0)d0$ 7

Figure 7: Solution to Section 15.7, problem 45

2. Section 15.8: Solutions, common mistakes and corrections:

V=X+4) $\overline{-0}Y, Y = -4$ (3X+2Y) (X+4Y) d Xdy $y = \frac{1}{10} (3V - 4)$ $\chi = \frac{1}{5} (24 - 4)$ $\frac{1}{\sqrt{2}} = -\frac{x}{4} + \frac{1}{\sqrt{2}} = -\frac{1}{4} \left(\frac{1}{5} (24 - 4) \right)$

Figure 8: Solution to Section 15.8, problem 7

 $\iint \left(\int \frac{y}{x} + \int xy \, dx \, dy \, (bounded by \, xy = 1, \, xy = 9, \, y = x \\ y = 4x, \, x = \frac{w}{v}, \, y = 4x \right)$ $\frac{\mu}{\nu} & \chi = \mu v = \frac{\gamma}{x} = v^2 & \chi \chi = \mu^2$ $\mathcal{J}(\mathcal{U}, \mathbf{v}) = \frac{\partial(\mathbf{x}, \mathbf{y})}{\partial(\mathcal{U}, \mathbf{v})} = \begin{vmatrix} \mathbf{v}^{-1} & -\mathcal{U}(\mathbf{v}^{-2}) \\ \mathbf{v} & \mathcal{U} \end{vmatrix} = \mathbf{v}^{-1}\mathcal{U} + \mathbf{v}^{-1}\mathcal{U} = \frac{\mathbf{x}\mathcal{U}}{\mathbf{v}}$ y = X = 2 $\frac{u}{\sqrt{2}} = uV = 2$ V = 1 j = 4X = 2 $\frac{4u}{\sqrt{2}} = uV = 2$ Xy = 1 = 2 u = 1 j = 2 Xy = 9 = 2 u = 3 $= \frac{3}{1} \int_{1}^{2} (v + u) \left(\frac{2u}{v}\right) dv du = \int_{1}^{3} \int_{1}^{2} 2u + \frac{2u}{v} dv du$ $= \int_{1}^{3} (2uv + 2u) \ln v \Big|_{1}^{2} du = \int_{1}^{3} (2u + 2u) \ln 2 du$ $= u^{2} + \frac{2}{3} u^{3} \ln 2 \Big|_{1}^{3} = 8 + \frac{52}{3} \ln 2 H$

Figure 9: Solution to Section 15.8, problem 9

19 Evaluate
$$\iiint |x|| |x|| |dx|| dy dz$$
 over the solid $\frac{y^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$
Let $x = au$ $y = by$ $z = cw$ $\iiint |au| (bu) (uu) |abc dud v dw $z = 53$
 $\frac{2x}{a^4} = a = \frac{2x}{a^4} = 0 = \frac{2x}{a^5} = 0$
 $\frac{2x}{a^5} = a = \frac{2x}{a^5} = 0 = \frac{2x}{a^5} = 0$
 $\frac{2x}{a^5} = 0 = \frac{2z}{a^5} = 0 = \frac{2z}{a^5} = 0$
 $\frac{2z}{a^5} = 0 = \frac{2z}{a^5} = 0 = \frac{2z}{a^5} = c$
 $8a^2b^2c^2 \int_{a}^{a} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} (psind cs6)(psind sin0)(pcc p)(psind chop 4p 40)$
 $= 8a^2b^2c^2 \int_{a}^{a} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} dp dp d0$
 $= 8a^2b^2c^2 \int_{a}^{a} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} dp dp d0$
 $= 8a^2b^2c^2 \int_{a}^{a} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} dp dp d0$
 $= 8a^2b^2c^2 \int_{a}^{a} \int_{a}^{b} \int_{a}^{b} \int_{a}^{b} dp dp d0$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (as 0 sin 0 ust (\frac{b}{b})) d0$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (as 0 sin 0 [\int_{a}^{a} (ab d)) d0$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (as 0 sin 0 d0)$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (ab 5c^2) \int_{a}^{a} (ab 0 sin 0 d0)$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (ab 0 sin 0 d0)$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (ab 0 sin 0 d0)$
 $= \frac{8}{b}a^2b^2c^2 \int_{a}^{a} (ab 0 sin 0 d0)$$

Figure 10: Solution to Section 15.8, problem 19

3. Homework 14, problem 3:

 $\begin{aligned} x = r\cos\theta, \ y = r\sin\theta, \\ x = r\sin\theta \cos\theta, \\ y = r\sin\theta, \\ z = r\cos\theta, \\ z = r\sin\theta, \\ z = r\cos\theta, \\ z = r\sin\theta, \\ z = r\cos\theta, \\ z = r\sin\theta, \\ z =$ 0650 -+ Sind U + Cujo 0 = Y (050 (050 Siho .05h

Figure 11: Solution to problem 3

4. Chap 15, Additional and Advanced Exercises:

Chap 15 add. $\frac{11}{2} \int_{0}^{\infty} \frac{e^{-ax} - e^{-bx}}{X} dx \left(\frac{e^{-ax} - e^{-bx}}{X} \right)_{a}^{b} e^{-xy} dy$ $= \int_{0}^{\infty} \int_{a}^{b} e^{-xy} dy dx = \int_{a}^{b} \int_{0}^{\infty} e^{-xy} dx dy$ $=\int_{a}^{b} \lim_{c\to\infty} \int_{0}^{c} e^{-xy} dx dy = \int_{a}^{b} \lim_{c\to\infty} -\frac{e^{-xy}}{y} \int_{0}^{c} dy$ $= \int_{a}^{b} \frac{1}{y} dy = \ln y \Big|_{a}^{b} \quad b f$ = lnb f ln a =ln b #

Figure 12: Solution to Chap 15, Additional and Advanced Exercises: problem 11