


41) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy = I^2$

$r^2 = x^2 + y^2$



$\rightarrow \int_0^{\pi/2} \lim_{b \rightarrow \infty} \int_0^b e^{-r^2} r dr d\theta = \int_0^{\pi/2} \lim_{b \rightarrow \infty} \left[-\frac{1}{2} e^{-r^2} \right]_0^b d\theta$

$\rightarrow \int_0^{\pi/2} \frac{1}{2} d\theta = \frac{1}{2} \theta \Big|_0^{\pi/2} = \frac{\pi}{4} = I^2$

$I = \int_0^\infty e^{-x^2} dx = \boxed{\frac{\sqrt{\pi}}{2}}$

b) $\lim_{x \rightarrow \infty} \text{erf} = \lim_{x \rightarrow \infty} \int_0^x \frac{2e^{-t^2}}{\sqrt{\pi}} dt$

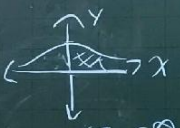
$\Rightarrow \int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$

$\left(\lim_{x \rightarrow \infty} \text{erf} = \frac{2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = 1 \right)$

Figure 3: Solution to Section 15.4, problem 41

15.4) 42.

$\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^2} dx dy$



$\int_0^{\pi/2} \int_0^\infty \frac{1}{1+r^2} r dr d\theta$

$\int_0^{\pi/2} \left(\lim_{b \rightarrow \infty} \int_0^b \frac{1}{1+r^2} r dr \right) d\theta$

$= \int_0^{\pi/2} \left(\lim_{b \rightarrow \infty} \left[\frac{1}{2} \ln(1+r^2) \right]_0^b \right) d\theta$

$= \int_0^{\pi/2} \frac{1}{2} d\theta$

$= \frac{1}{2} \theta \Big|_0^{\pi/2} = \boxed{\frac{\pi}{4}}$




Figure 4: Solution to Section 15.4, problem 42

2. Homework 13, problem 3:

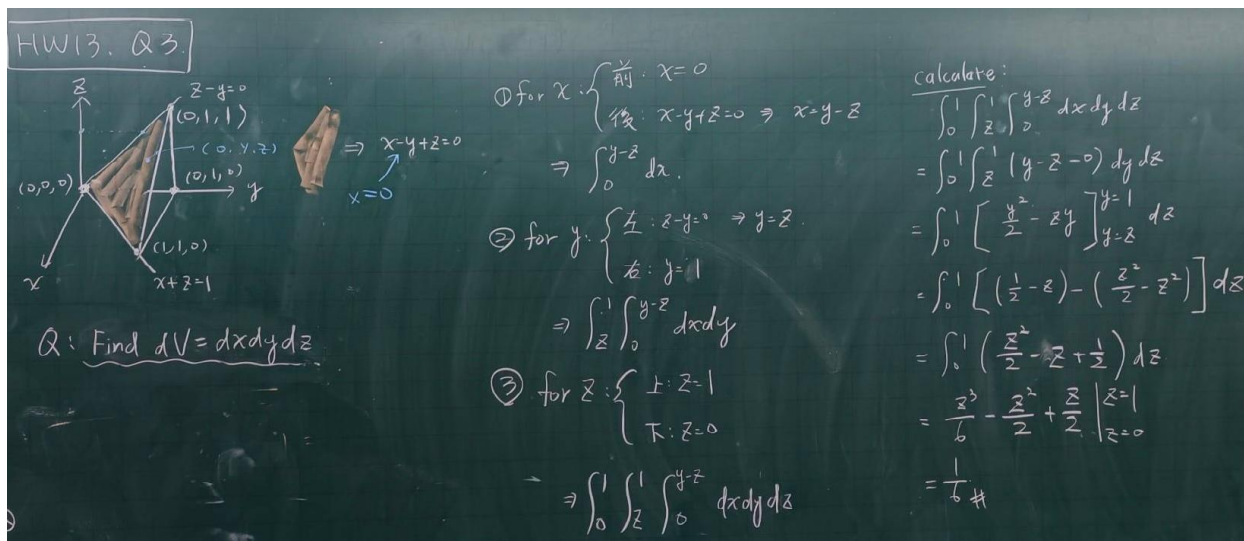


Figure 5: Solution to problem 3

3. Section 15.5: Solutions, common mistakes and corrections:

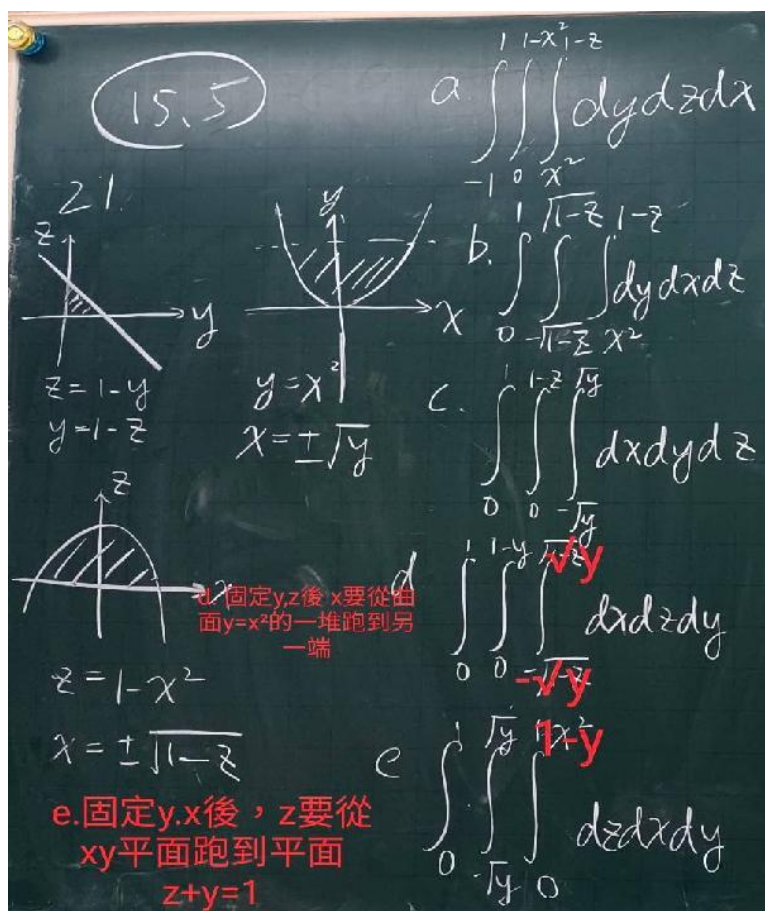



Figure 6: Solution to Section 15.5, problem 21

15.5
41.



$$\int_0^4 \int_0^1 \int_0^{2y} \frac{4 \cos(x^2)}{2\sqrt{z}} dx dy dz$$

$$= \int_0^4 \int_0^2 \int_0^{\frac{x}{2}} \frac{4 \cos(x^2)}{2\sqrt{z}} dy dx dz$$

$$= \int_0^4 \frac{1}{4\sqrt{z}} \int_0^2 4x \cos(x^2) dz dx$$

$u = x^2, du = 2x dx$
 $\cos u \cdot 2 du$

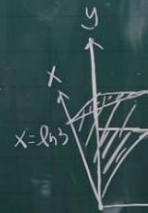
$$= \int_0^4 \frac{1}{4\sqrt{z}} (2 \sin u - \int_0^2) dz$$

$$= \frac{1}{4} \int_0^4 z^{-\frac{1}{2}} (2 \sin 4) dz$$

$$= 2 \sin 4$$

Figure 7: Solution to Section 15.5, problem 41

43.



$$\int_0^1 \int_{\sqrt{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2} dx dy dz$$

$$= \int_{x=0}^{\ln 3} \int_{y=0}^1 \int_{z=0}^{y^2} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2} dz dy dx$$

$$= \int_{x=0}^{\ln 3} \int_{y=0}^1 \pi e^{2x} y \sin(\pi y^2) dy dx$$

$$= \int_{x=0}^{\ln 3} \left. -\frac{e^{2x}}{2} \cos(\pi y^2) \right|_0^1 dx$$

$$= \int_{x=0}^{\ln 3} e^{2x} dx$$

$$= \left. \frac{e^{2x}}{2} \right|_0^{\ln 3} = 4$$

$y = \sqrt{z} \Leftrightarrow z = y^2$
 $(1,1)$

Figure 8: Solution to Section 15.5, problem 43