# Brief solutions to selected problems in homework 12

1. Section 14.10: Solutions, common mistakes and corrections:

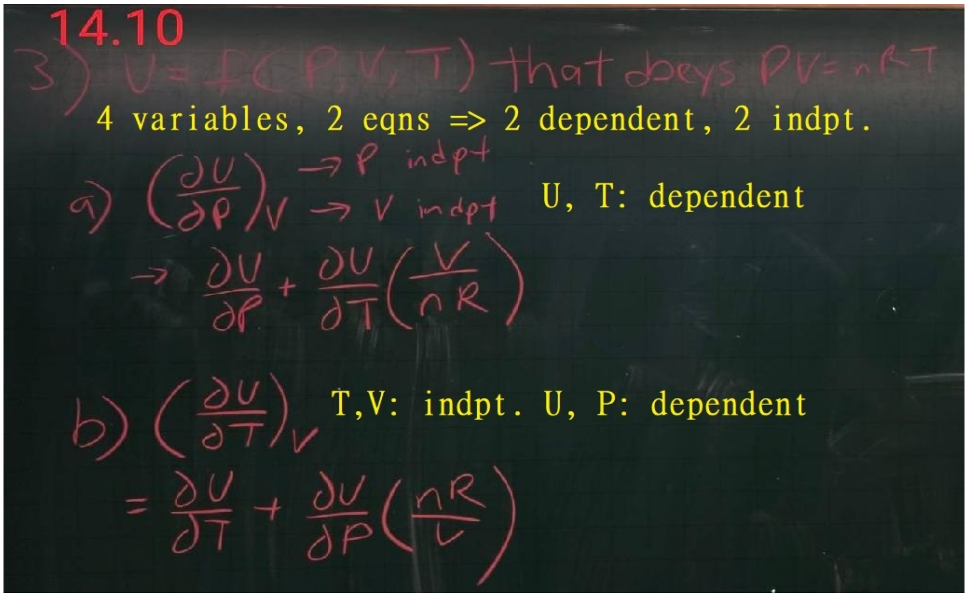


Figure 1: Solution to Section 14.10, problem 3

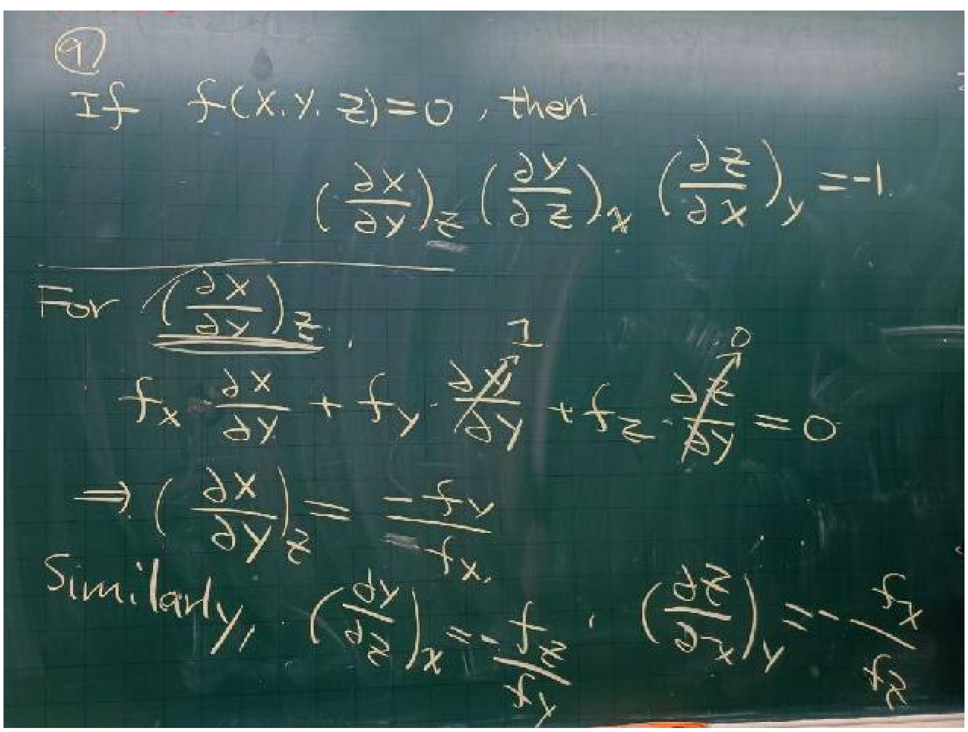


Figure 2: Solution to Section 14.10, problem 9

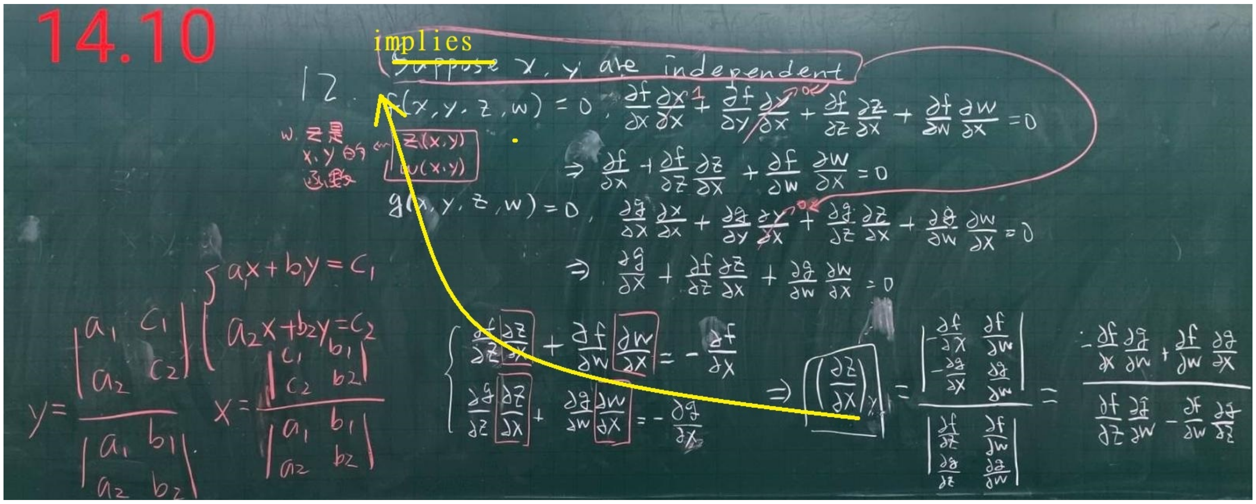


Figure 3: Solution to Section 14.10, problem 12

2. Homework 12, problem 2:

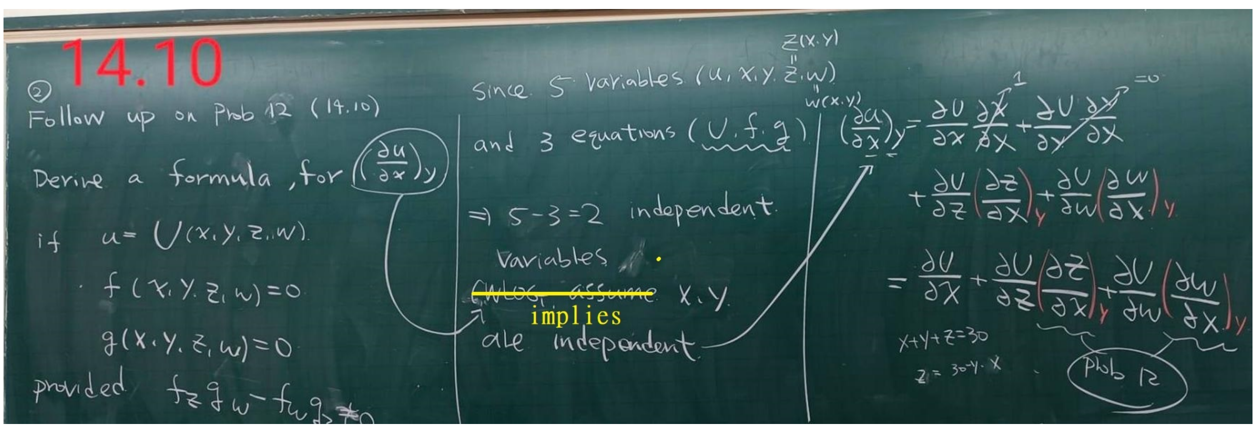


Figure 4: Solution to problem 2

3. Section 15.1: Solutions, common mistakes and corrections:

   Problem 36:

   Since $f(x,y)$ is continuous in $(x,y)$, it is also continuous in $x$ for fixed $y$, and also continuous in $y$ for fixed $x$. In addition $\int_c^y f(u,v)dv$ is also continuous in $u$ for fixed $y$.

   From Fundamental Theorem of Calculus:
   $\frac{d}{dx}\int_a^x g(u)du = g(x)$ provide $g(x)$ is continuous.

   It follows that $F_x(x,y) = \frac{d}{dx}\int_a^x \left(\int_c^y f(u,v)dv\right) du = \int_c^y f(x,v)dv$. Therefore $F_{xy}(x,y) = \frac{d}{dy}F_x(x,y) = \frac{d}{dy}\int_c^y f(x,v)dv = f(x,y)$.

   From Fubini's Theorem, $F(x,y) = \int_c^y \left(\int_a^x f(u,v)du\right) dv$. Using similar calculation as above, we also have $F_{yx}(x,y) = \frac{d}{dx}F_y(x,y) = \frac{d}{dx}\int_a^x f(u,y)du = f(x,y)$.

4. Section 15.2: Solutions, common mistakes and corrections:

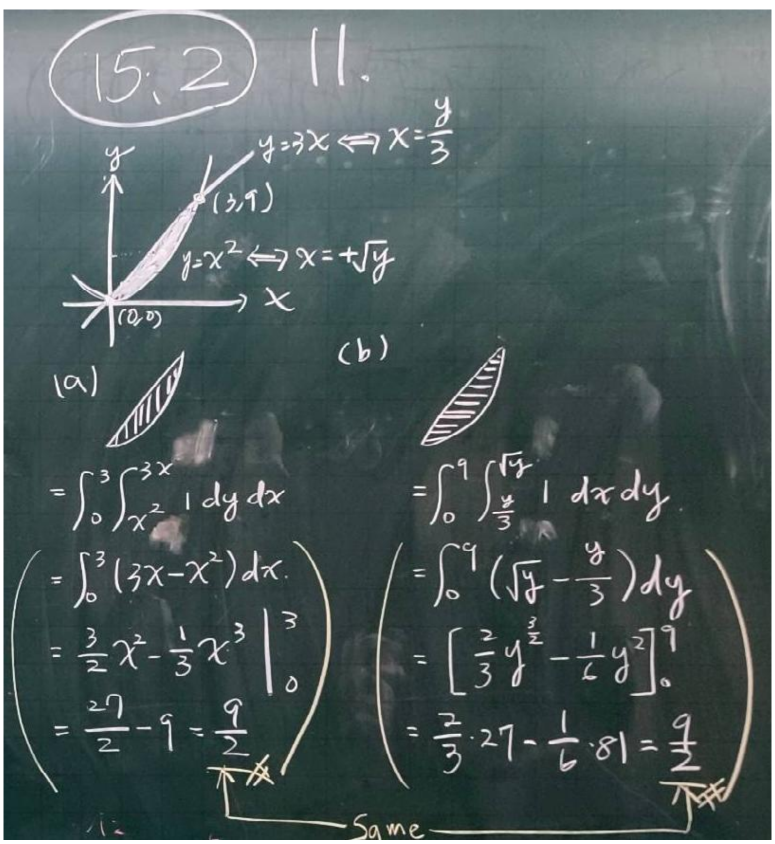


Figure 5: Solution to Section 15.2, problem 11

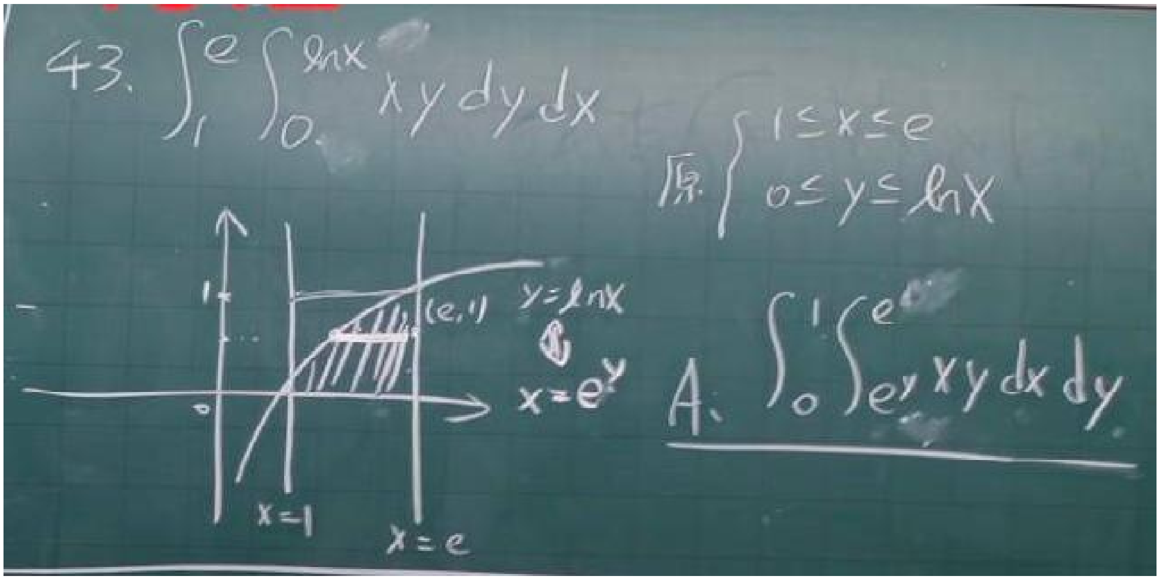


Figure 6: Solution to Section 15.2, problem 43

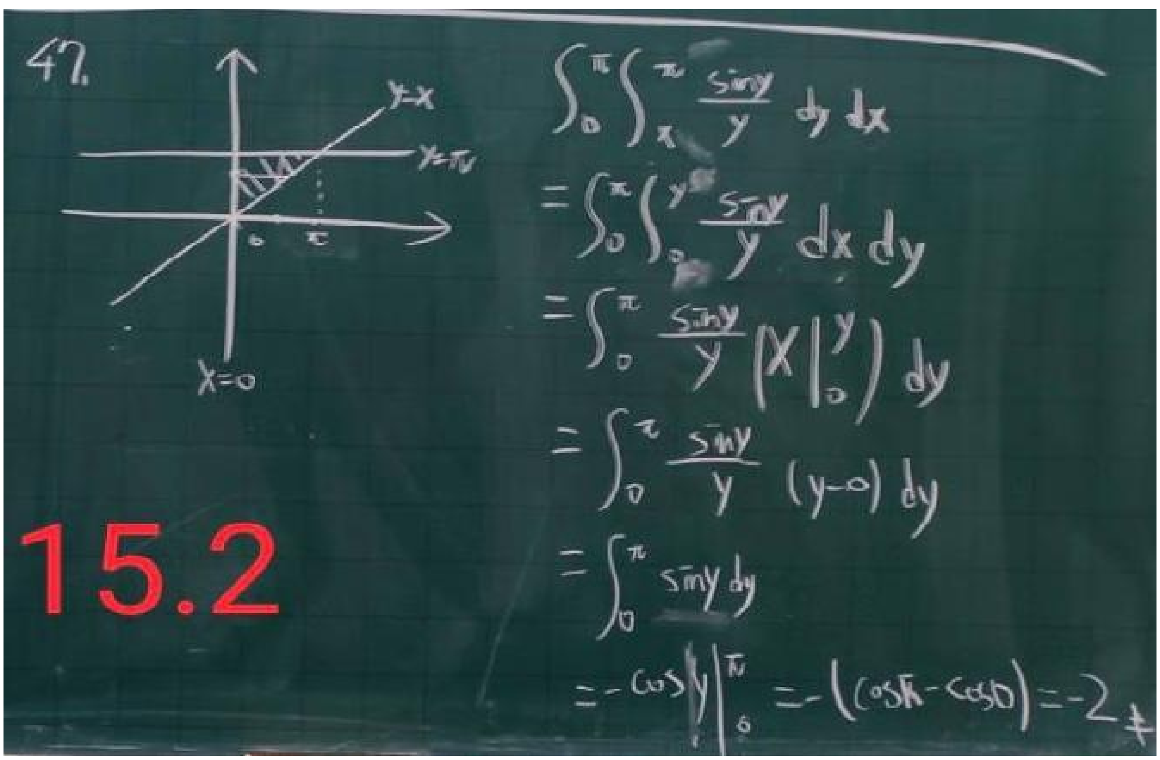


Figure 7: Solution to Section 15.2, problem 47