

Brief solutions to selected problems in homework 11

1. Section 14.8: Solutions, common mistakes and corrections:

39. $f(x, y, z) = (x-0)^2 + (y-0)^2 + (z-0)^2$
 $= x^2 + y^2 + z^2$
 $g(y, z) = y + 2z - 12$
 $t(x, y) = x + y - 6$
 $\nabla f = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$
 $\nabla g = \mathbf{j} + 2\mathbf{k}$
 $\nabla t = \mathbf{i} + \mathbf{j}$
 $\nabla f = \lambda \nabla g + \beta \nabla t$
 $2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k} = \lambda(\mathbf{j} + 2\mathbf{k}) + \beta(\mathbf{i} + \mathbf{j})$
 $2x\mathbf{i} = \beta\mathbf{i} \Rightarrow x = \frac{\beta}{2}$
 $2y\mathbf{j} = (\lambda + \beta)\mathbf{j} \Rightarrow y = \frac{\lambda + \beta}{2}$
 $2z\mathbf{k} = 2\lambda\mathbf{k} \Rightarrow z = \lambda$
 $t(x, y) = \frac{\beta}{2} + \frac{\lambda + \beta}{2} - 6 = 0 \Rightarrow \lambda + \beta = 12 \dots \textcircled{1}$
 $g(y, z) = \frac{\lambda + \beta}{2} + 2\lambda - 12 = 0 \Rightarrow \lambda + \beta + 4\lambda = 24 \Rightarrow 5\lambda + \beta = 24 \dots \textcircled{2}$
 $2 \times \textcircled{2} - \textcircled{1} \Rightarrow 9\lambda = 12 \Rightarrow \lambda = \frac{4}{3}$
 $\beta = 12 - \lambda = 12 - \frac{4}{3} = \frac{32}{3}$
 $x = \frac{\beta}{2} = \frac{16}{3}$
 $y = \frac{\lambda + \beta}{2} = \frac{\frac{4}{3} + \frac{32}{3}}{2} = \frac{36}{6} = 6$
 $z = \lambda = \frac{4}{3}$
 $A: (\frac{16}{3}, 6, \frac{4}{3})$

Figure 1: Solution to Section 14.8, problem 39

2. Section 14.9: Solutions, common mistakes and corrections:

9. $f(x, y) = \frac{1}{1-x-y} = (1-x-y)^{-1}$
 \because coefficient of x & y are the same
 $\Rightarrow f_x = f_y = (1-x-y)^{-2}$
 $\Rightarrow f_{xx} = f_{xy} = f_{yy} = \frac{2}{(1-x-y)^3}$
 $\Rightarrow f_{xxx} = f_{xxy} = f_{xyy} = f_{yyy} = \frac{6}{(1-x-y)^4}$
 quadratic approximation
 $\Rightarrow f(x, y) \approx 1 + x + y + \frac{1}{2}(2x^2 + 4xy + 2y^2)$
 $= 1 + (x+y) + (x+y)^2 \#$
 cubic approximation $\Rightarrow f(x, y) \approx 1 + (x+y) + (x+y)^2 + \frac{1}{6}(x+y)^3$
 $= 1 + (x+y) + (x+y)^2 + (x+y)^3 \#$

Figure 2: Solution to Section 14.9, problem 9

similarly here
missing ! !

$$f_{xxx}, f_{xxy}, f_{xyy}, f_{yyy} \leq \left(\frac{1}{6}(0)^3 + 3 \cdot (0.1)^3 + 3 \cdot (0.1)^3 + (0.1)^3 \right)$$

$$f_x = -\cos y \sin x = 0$$

$$f_y = -\cos x \sin y = 0$$

$$f_{xx} = -\cos y \cos x = -1$$

$$f_{yy} = -\cos x \cos y = -1$$

$$f_{xy} = \sin x \sin y = 0$$

$$f \approx \left[f(0,0) + x f_x(0,0) + y f_y(0,0) \right] + \frac{1}{2} \left(x^2 f_{xx}(0,0) + y^2 f_{yy}(0,0) + 2xy f_{xy}(0,0) \right)$$

$$|E| \leq \frac{1}{(2+1)!} \left(|x|^3 |f_{xxx}| + 3|x|^2 |y| |f_{xxy}| + 3|x| |y|^2 |f_{xyy}| + |y|^3 |f_{yyy}| \right)$$

Figure 3: Solution to Section 14.9, problem 11