Brief solutions to selected problems in homework 09

1. Section 14.4: Solutions, common mistakes and corrections:

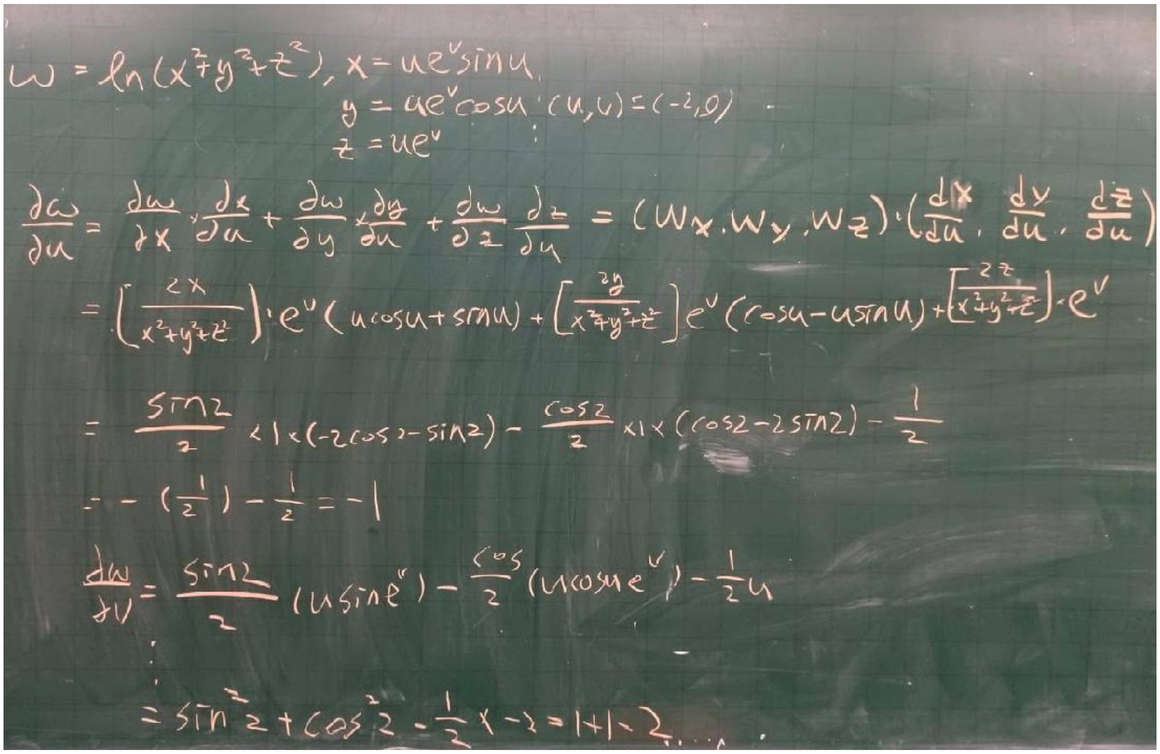

Figure 1: Solution to Section 14.4, problem 10, method 1

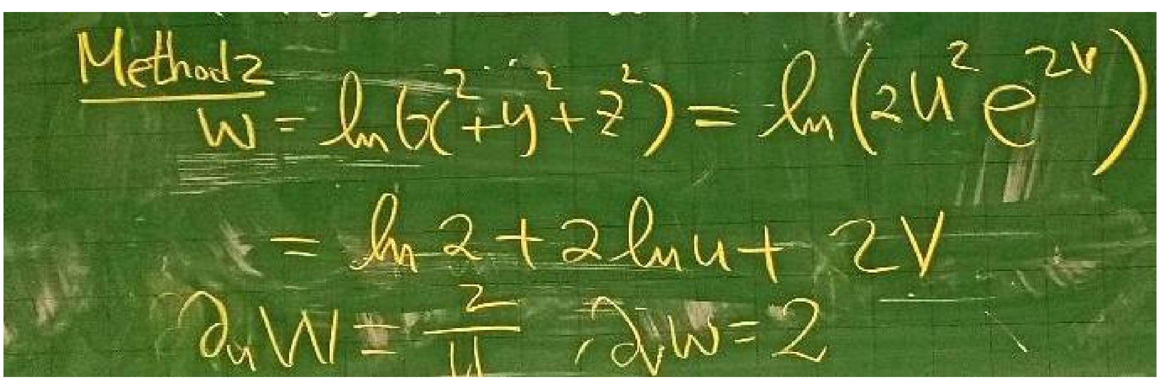

Figure 2: Solution to Section 14.4, problem 10, method 2

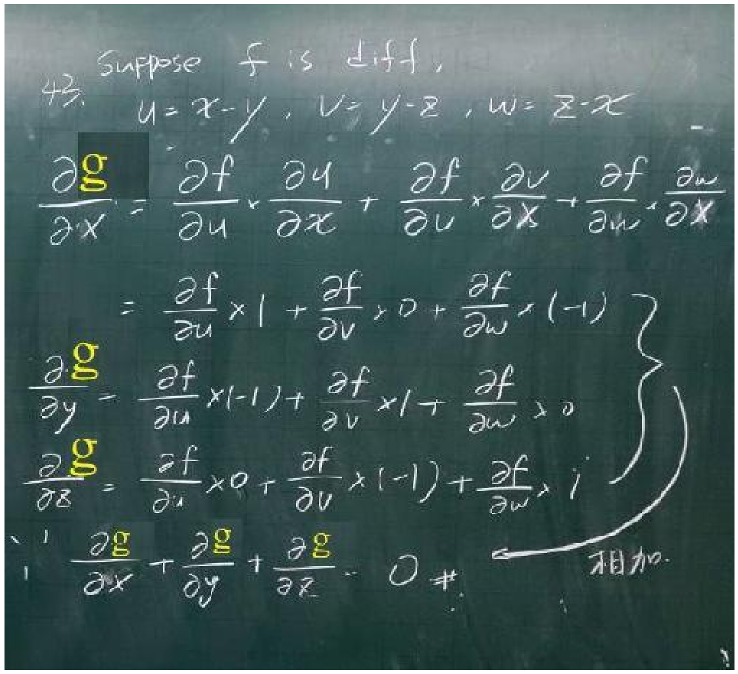

Figure 3: Solution to Section 14.4, problem 43

Remark: There may be some confusion in the statement of Problem 43. It should have been written as following:

let $f(u, v, w)$ be differentiable and $g(x, y, z) = f(u, v, w)$ where $u = x - y$, $v = y - z$, $w = z - x$. Show that $\dfrac{\partial g}{\partial x} + \dfrac{\partial g}{\partial y} + \dfrac{\partial g}{\partial z} = 0$

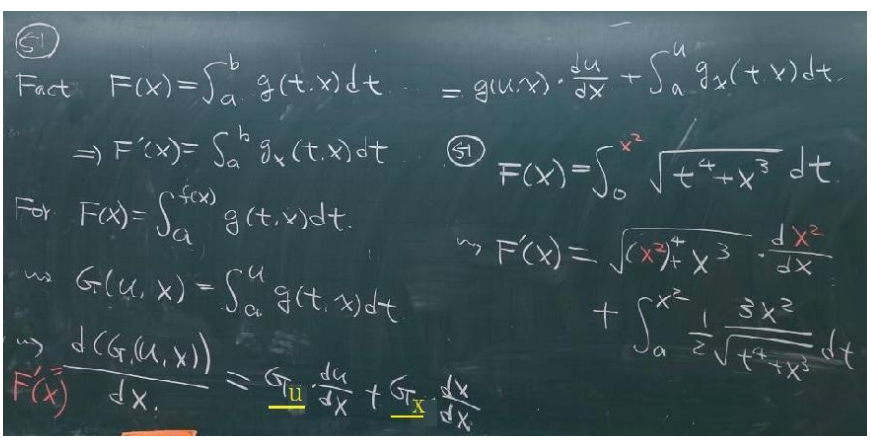

Figure 4: Solution to Section 14.4, problem 51

Remark: The meaning of $G_u \cdot \dfrac{du}{dx} + G_x \cdot \dfrac{dx}{dx}$ is quite clear in the context. Alternatively, one could write it as $\partial_1 G \cdot \dfrac{du}{dx} + \partial_2 G \cdot \dfrac{dx}{dx}$ to avoid potential confusion about what $G_x(u(x), x)$ really means.

2. Section 14.5: Solutions, common mistakes and corrections:

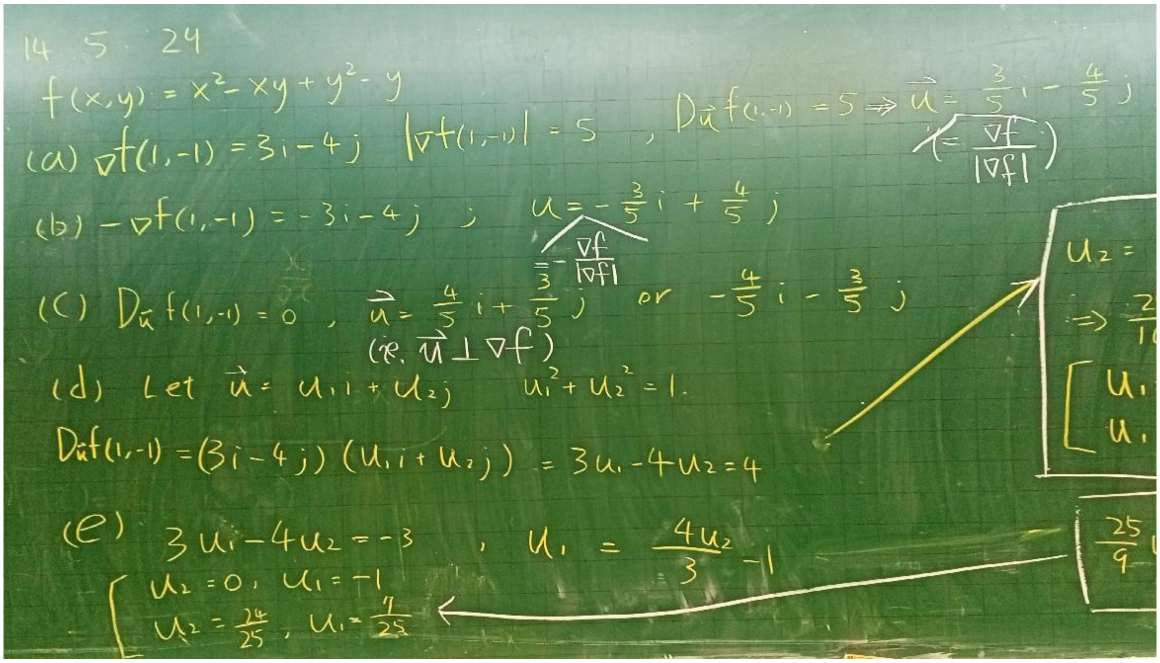

Figure 5: Solution to Section 14.5, problem 29

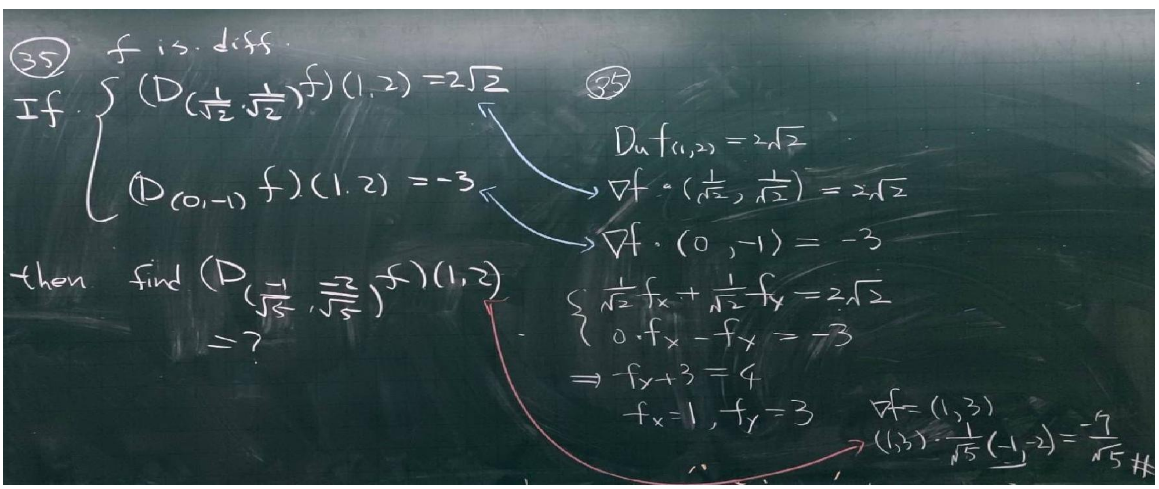

Figure 6: Solution to Section 14.5, problem 35

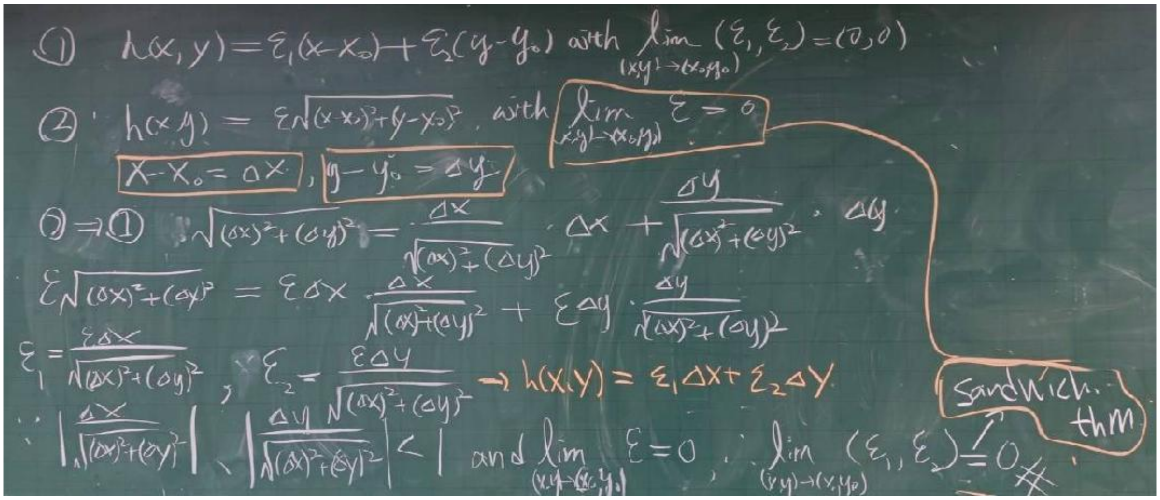

Figure 7: Solution to Homework 08, problem 2, part 1

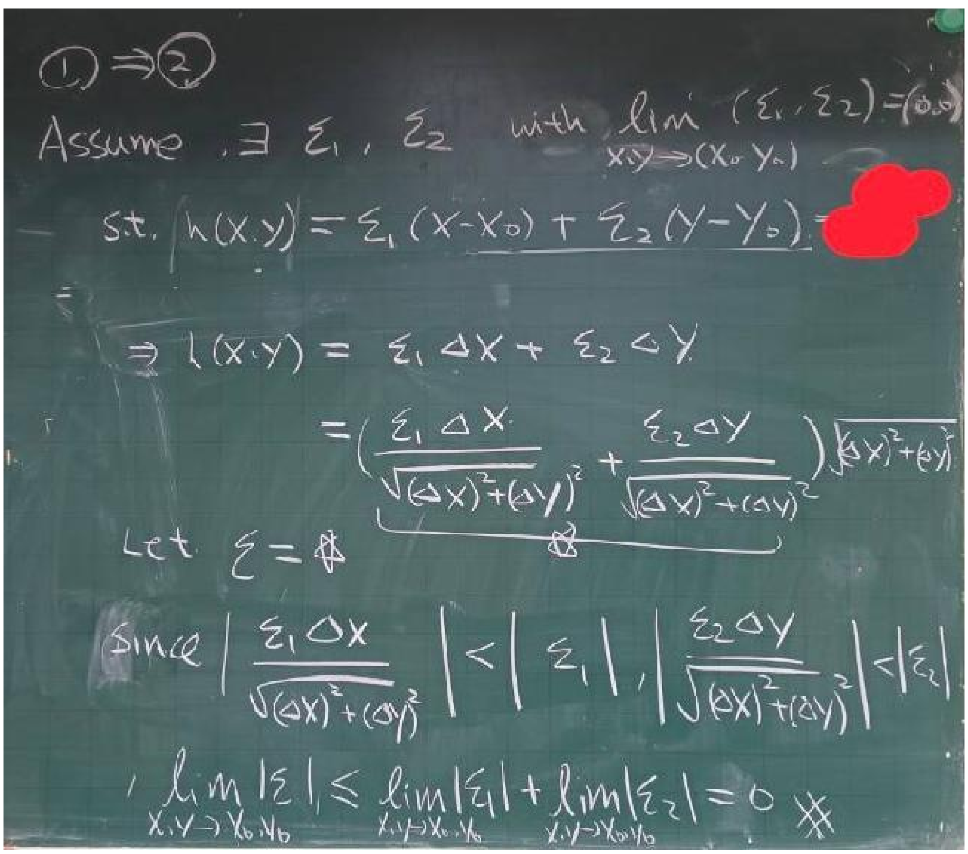

Figure 8: Solution to Homework 08, problem 2, part 2

3. Problem 3:

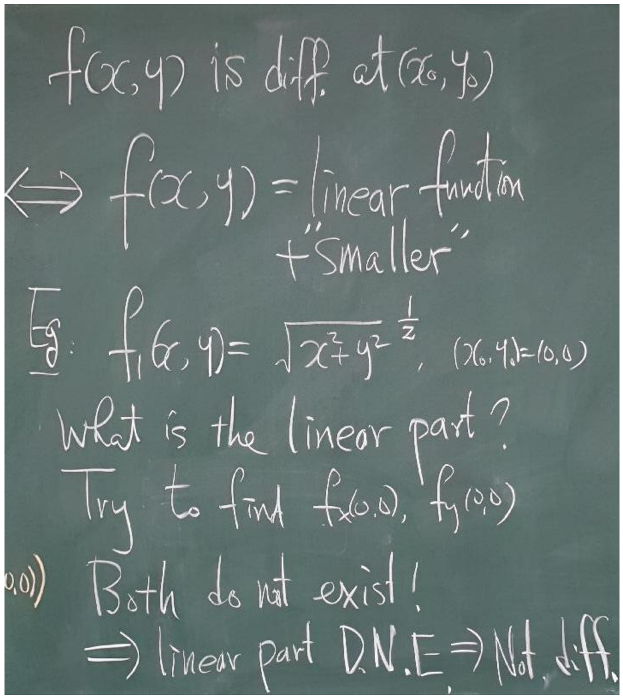

Figure 9: Problem 3

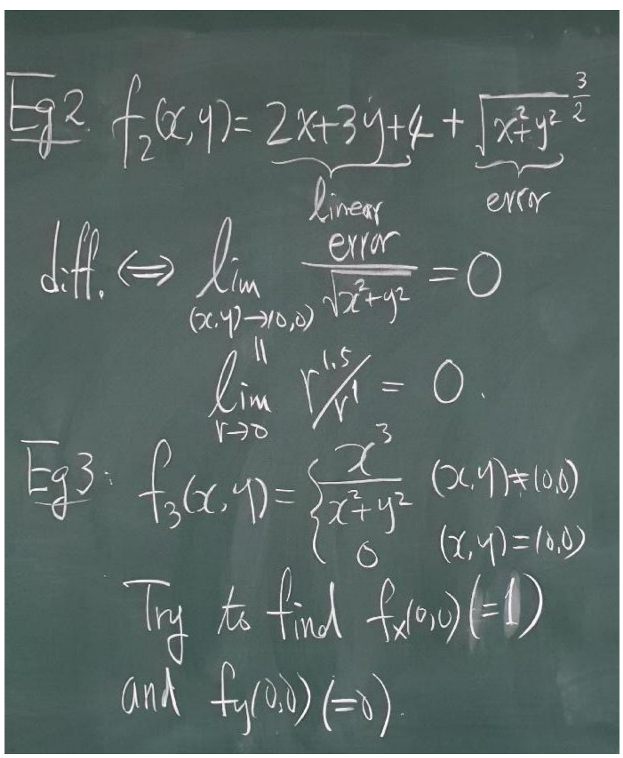

Figure 10: Problem 3, continued

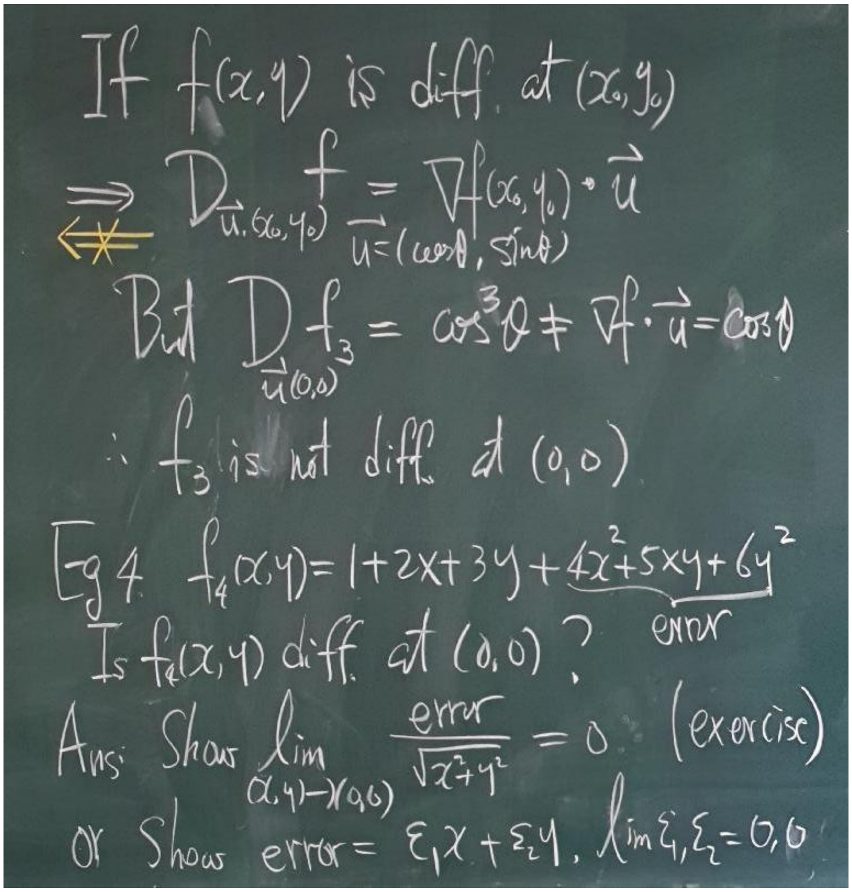

Figure 11: Problem 3, continued

4. (Extra credit, very hard!) True or False?

   If $f_x(0,0)$, $f_y(0,0)$ and $D_{(\cos\theta,\sin\theta),(0,0)}f$ all exist and

   $$D_{(\cos\theta,\sin\theta),(0,0)}f = f_x(0,0)\cos\theta + f_y(0,0)\sin\theta, \text{ for all } \theta \in [0, 2\pi],$$

   then f is differentiable at $(0,0)$.