# Brief solutions to selected problems in homework 07

1. Section 14.2: Solutions, common mistakes and corrections:

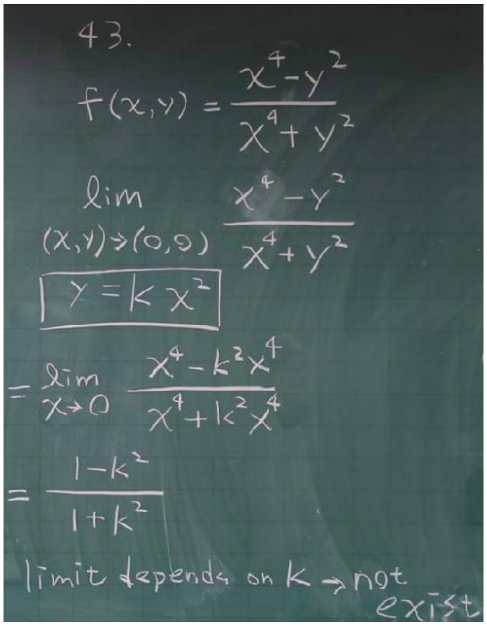


Figure 1: Solution to Section 14.2, problem 43

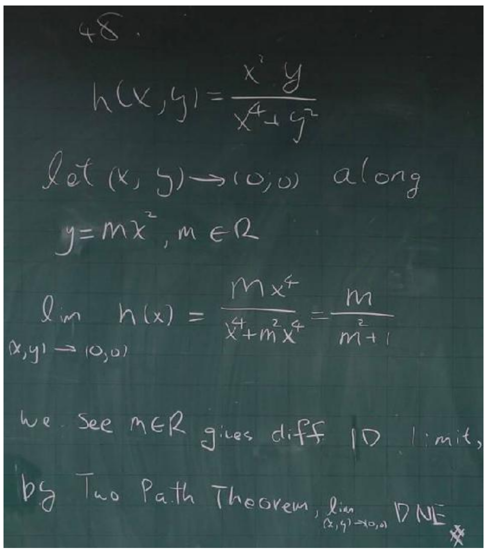


Figure 2: Solution to Section 14.2, problem 48

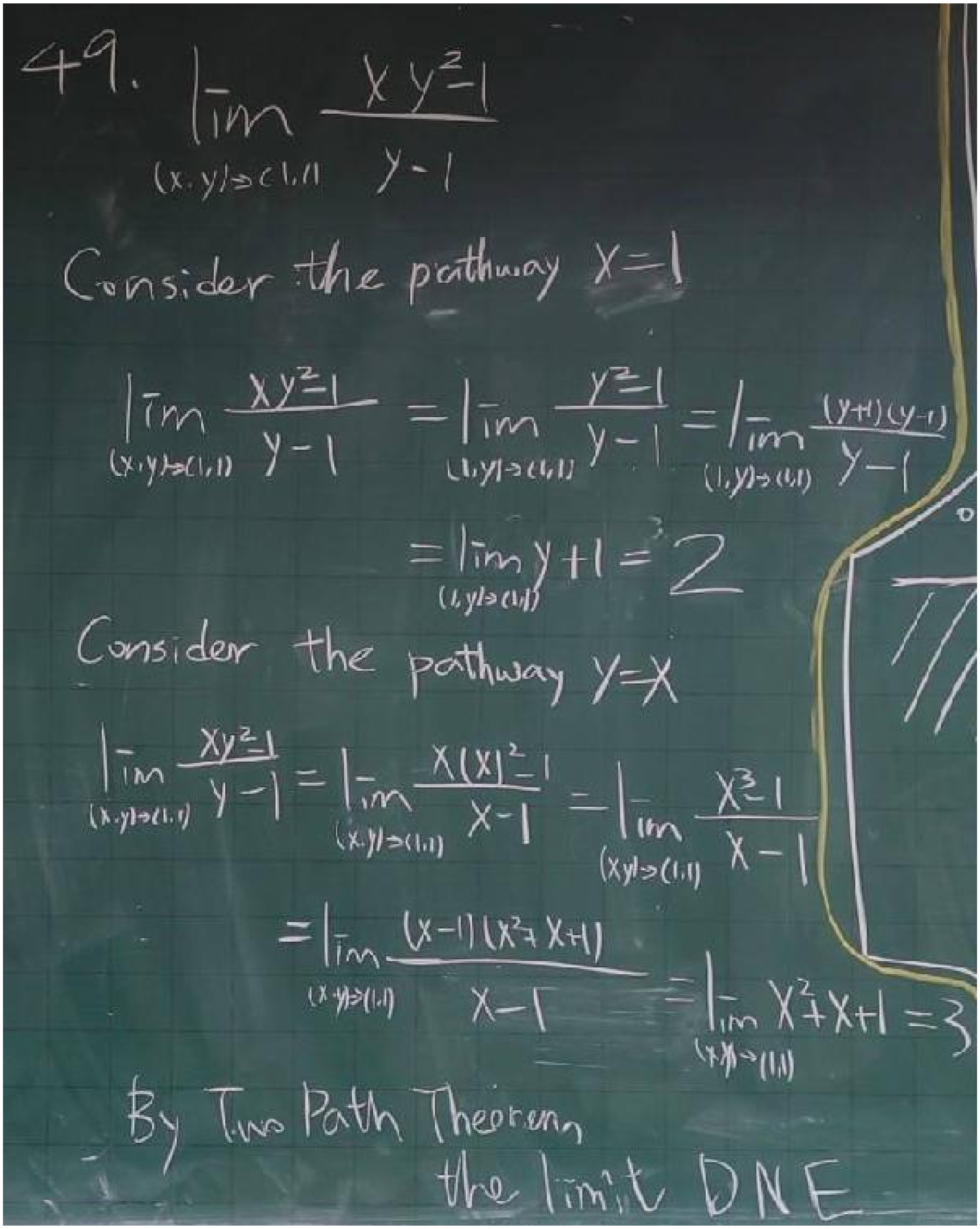


Figure 3: Solution to Section 14.2, problem 49

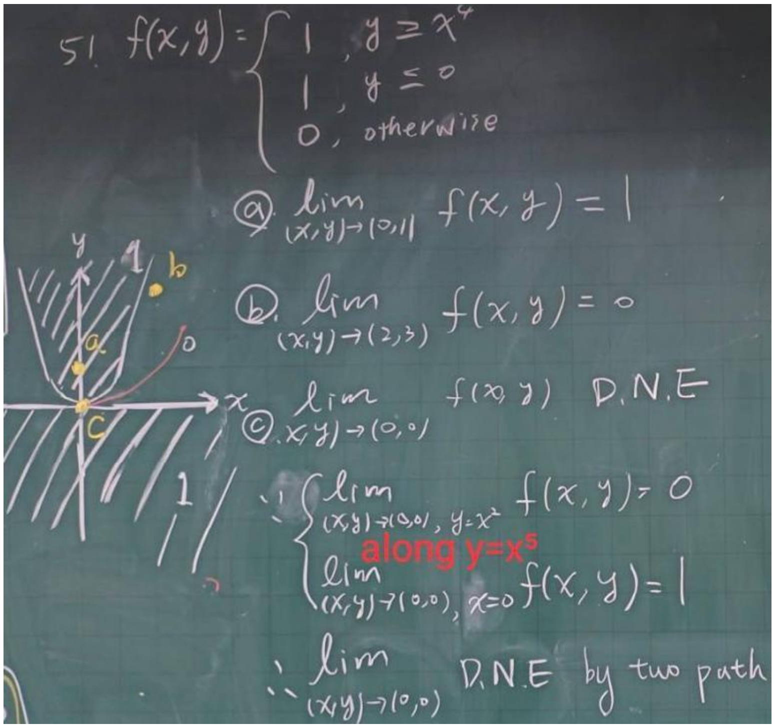


Figure 4: Solution to Section 14.2, problem 51

**Remark**: The Two Path Test can only be used to show the limit does not exist. If the two limits along two different paths are the same, it is not enough to say the 2D limit is the same as the limit along the two different paths.

To show the limit exists (and equals $L$), one needs to check that the values of $f(x, y)$ inside $0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$ to be close enough to $L$. The key here is to find an appropriate $\delta > 0$ for each $\varepsilon > 0$.

In problem (a), $\delta = \frac{1}{2}$ will work for any $\varepsilon > 0$ since

$$\{(x, y), \quad 0 < \sqrt{(x - 0)^2 + (y - 1)^2} < \frac{1}{2}\} \subset \{(x, y), y \geq x^4\}$$

Since if $(x, y)$ is in the first set, then $|x| < \frac{1}{2}$, $y - 1 > -\frac{1}{2}$, so $y > \frac{1}{2} > (\frac{1}{2})^4 > x^4$), therefore $(x, y)$ is in the second set.

Similarly, it is easy to check that $\delta = 1$ (or smaller) works for problem (b).

For problem (c), one can apply the Two Path Test by checking the limits along the two paths "$y = 2x^4$ and $y = \frac{1}{2}x^4$" (or along "$y = 0$ and $y = x^5$" as shown in the picture).

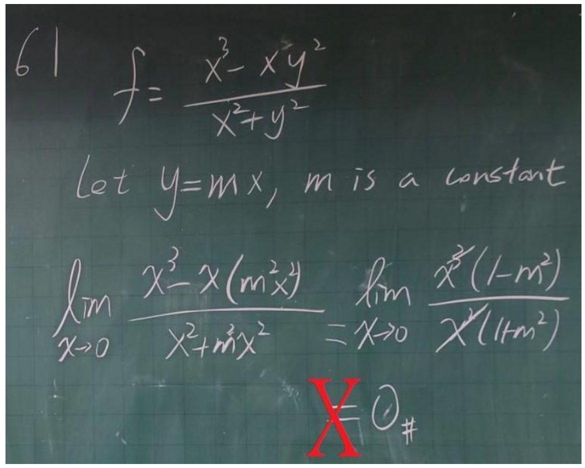


Figure 5: Solution to Section 14.2, problem 61

**Remark**: If the limits along all paths of the form $y = mx$, $m \in \mathbb{R}$ are the same, it is not enough to say the 2D limit equals this limit. See problem 63 below for correct solution method.

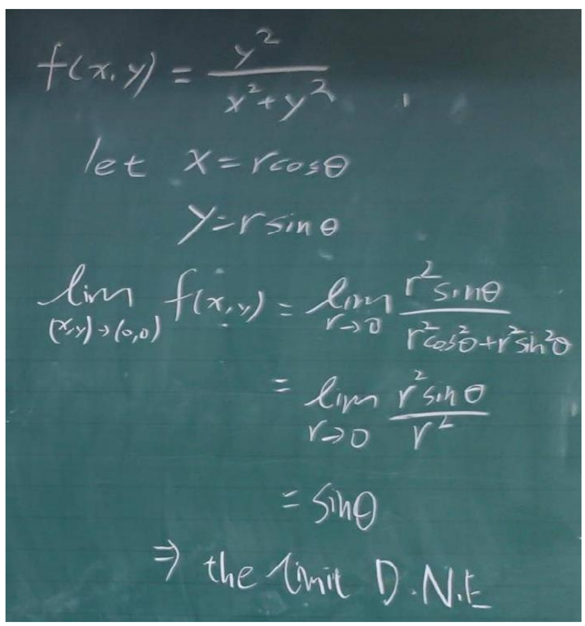


Figure 6: Solution to Section 14.2, problem 63