

tion10.7

59. The series $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots$

題目

(a) $\int \tan x dx = \ln|\sec x| + C$

$\int (x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots) dx$

$\int x dx + \int \frac{x^3}{3} dx + \dots$

$= \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + \frac{31x^{10}}{14175} + \dots$

Converges in $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(b) $\sec^2 x = \frac{d}{dx}(\tan x)$

$= \frac{d}{dx}(x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots)$

$= \frac{d}{dx}(x) + \frac{d}{dx}(\frac{x^3}{3}) + \frac{d}{dx}(\frac{2x^5}{15}) + \dots$

$= 1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + \frac{62x^8}{315} + \dots$

Converges $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Figure 3: Solution to Section 10.7, problem 57

Alternatively, instead of using indefinite integral with an undetermined constant C , it is simpler to use definite integral with the center of the power series ("0") to be the lower limit of integration: $\ln|\sec x| - \ln|\sec 0| = \int_0^x \tan t dt = \int_0^x t + \frac{t^3}{3} + \frac{2t^5}{15} + \dots dt$.

sec10.7

60. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ conv on $(-1, 1)$

$\sum_{n=0}^{\infty} \frac{n^2}{2^n}$, $(x = \frac{1}{2}) \in (-1, 1)$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$ on $(-1, 1)$

$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \dots$

$\frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = \frac{x+1}{(1-x)^3} = 1 + 4x + 9x^2 + 16x^3 + \dots$

$\frac{x(x+1)}{(1-x)^3} = x + 4x^2 + 9x^3 + 16x^4 + \dots$ $x = \frac{1}{2} \Rightarrow \frac{1}{2} + \frac{4}{2^2} + \frac{9}{2^3} + \dots = \sum_{n=0}^{\infty} \frac{(n+1)^2}{2^{n+1}}$

$= 6$

Figure 4: Solution to Section 10.7, problem 60

2. Section 10.8: Solutions, common mistakes and corrections:

10.8

5. $f(x) = x^{-1}$, $f'(x) = -x^{-2}$, $f''(x) = 2x^{-3}$, $f'''(x) = -6x^{-4}$

① $\boxed{a=2}$

$P_0(x) = \frac{1}{2}$

$P_1(x) = \frac{1}{2} - \frac{1}{4}(x-2)$

$P_2(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2$

$P_3(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$

$\frac{1+x+x^2+\dots}{1-x} \text{ if } |x| < 1$

② $f(x) = \frac{1}{x}$

$f''(x) = \frac{1}{2+(x-2)}$

$= \frac{1}{2} \frac{1}{1 + \frac{(x-2)}{2}}$

$= \frac{1}{2} \frac{1}{1 - \frac{(2-x)}{2}}$

$\rightarrow \text{let } \left| \frac{2-x}{2} \right| < 1$

$\Rightarrow f(x) = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{2-x}{2} \right)^n$

Figure 5: Solution to Section 10.8, problem 5

sec 10.8

35

$f(x) = (\sin x)(\ln(1+x))$

$(\sin x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$) A
conv. on \mathbb{R}

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$) B
conv. on $(-1, 1)$

$(\sin x) \ln(1+x) = AB$

$= \sum_{n=0}^{\infty} C_n X^n$

$\Rightarrow T_{f(x), 0} = AB = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots$
Thm. conv. on $(-1, 1)$

Figure 6: Solution to Section 10.8, problem 35