

Brief solutions to selected problems in homework 04

1. Section 10.3: Solutions, common mistakes and corrections:

Let $a_n = \int_n^\infty \frac{1}{x^{1.1}} dx$.

Goal: Find $N \in \mathbb{N}$

Set $a_N = \int_N^\infty \frac{1}{x^{1.1}} dx \leq 10^{-5}$

$\Rightarrow 10^{60} \leq N$

Figure 1: Solution to Section 10.3, problem 51

2. Section 10.6: Solutions, common mistakes and corrections:

28

$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n} \rightarrow a_n$

① $a_n > 0$

② $a_{n+1} < a_n$

$\lim_{n \rightarrow \infty} \frac{1}{n \ln n} = 0$

series

Alternating test \Rightarrow conv.

但加绝对值 $\sum_{n=2}^{\infty} \left(\frac{1}{n \ln n} \right)$ (div) $\hookrightarrow (p=1)$

\Rightarrow conditional conv. #

$\int_2^\infty \frac{1}{x \ln x} dx = \int_{\ln 2}^\infty \frac{du}{u} \quad \ln x = u$

Figure 2: Solution to Section 10.6, problem 28

(29) $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1}(n)}{n^2+1} = a_n$

let $f(x) = \frac{\tan^{-1}x}{1+x^2}$ where
 $f(n) = a_n, \forall n \in \mathbb{N}$
 $f'(x) = \frac{1 - (\tan^{-1}x)(2x)}{(1+x^2)^2} > 0$

check: a_n is positive, $a_{n+1} \leq a_n$

$\lim_{n \rightarrow \infty} \frac{\tan^{-1}n}{n^2+1} = \lim_{n \rightarrow \infty} \frac{\frac{\pi}{2}}{n^2+1} = 0$

Integral Test
 $\Rightarrow \int_1^{\infty} \frac{\tan^{-1}x}{x^2+1} dx = \lim_{b \rightarrow \infty} \left[\frac{1}{2} (\tan^{-1}(x))^2 \right]_1^b$

$= \lim_{b \rightarrow \infty} \frac{1}{2} (\tan^{-1}(b))^2 - \frac{1}{2} (\tan^{-1}(1))^2$
 $= \frac{1}{2} \left(\frac{\pi}{2}\right)^2 - \frac{1}{2} \left(\frac{\pi}{4}\right)^2$

\Rightarrow Absolute convergence

Figure 3: Solution to Section 10.6, problem 29

(49) $\left| \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} - \sum_{n=1}^4 (-1)^{n+1} \frac{1}{n} \right| \leq \boxed{?}$

(1) $a_n > 0$ for all n
 (2) $a_n > a_{n+1}$ for all n
 (3) $\lim_{n \rightarrow \infty} a_n = 0$

$|L - S_4| \leq \left| (-1)^{5+1} \frac{1}{5} \right| = 0.2$

$|L - S_m| \leq \left| (-1)^{m+2} \frac{1}{m+1} \right|$
 $= \left| \sum_{n=1}^m (-1)^{n+1} \frac{1}{n} \right|$

Figure 4: Solution to Section 10.6, problem 49

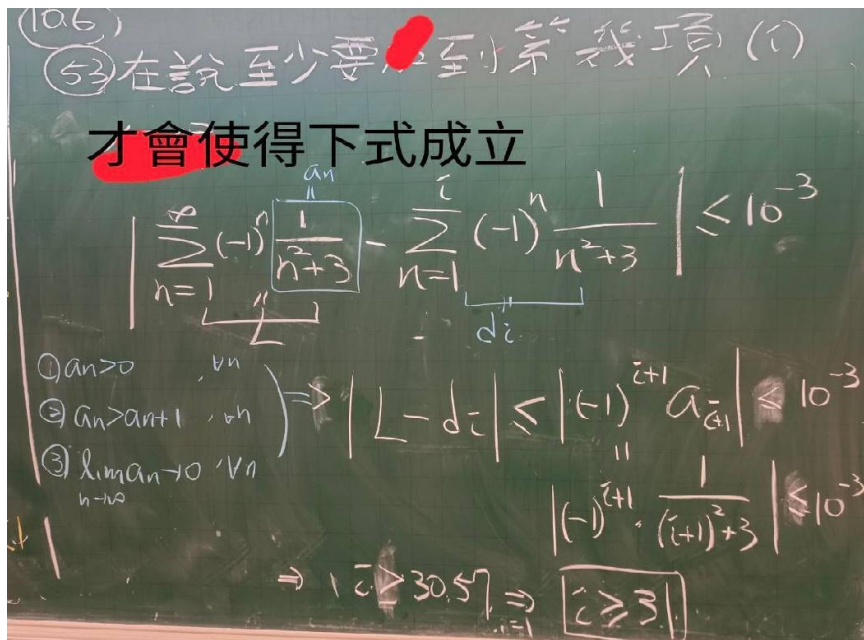


Figure 5: Solution to Section 10.6, problem 53

3. Section 10.7: Solutions, common mistakes and corrections:

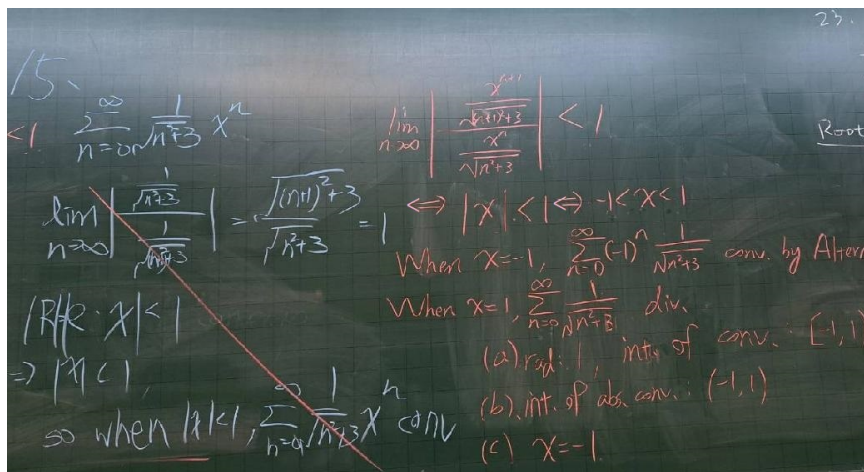


Figure 6: Solution to Section 10.7, problem 15

10.7 19

$$\sum_{n=0}^{\infty} \frac{1^n x^n}{3^n}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{1^{n+1} x^{n+1}}{3^{n+1}}}{\frac{1^n x^n}{3^n}} \right| = \left| \frac{1 \cdot x}{1 \cdot 3} \right|$$

$$\lim_{n \rightarrow \infty} \left| \frac{1 \cdot x}{1 \cdot 3} \right| = \left| \frac{x}{3} \right| = r$$

$r < 1 \rightarrow \left| \frac{x}{3} \right| < 1 \Rightarrow 3 < x < 3$

if $x=3 \rightarrow \sum_{n=0}^{\infty} 1^n$ - div.

$x=-3 \rightarrow \sum_{n=0}^{\infty} (-1)^n 1^n$ - div.

$A: -3 < x < 3$ not conditionally

1. $r_n = n > 0$
2. $f(x) = \sqrt{x} \cdot f'(x) > 0$

Figure 7: Solution to Section 10.7, problem 19

23.

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n$$

Root $\rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n |x| = |x| < 1 \Leftrightarrow -1 < x < 1$

(a) Radius = 1, int. of conv. (-1, 1)
(b) conv. (abs) (-1, 1)
(c) No such x.

When $x=1$, by nth-Term Test, $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$ div.
When $x=-1$, by nth-Term Test, $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$ div.

Figure 8: Solution to Section 10.7, problem 23

4. Homework 04, problem 4:

See page 7 of Lecture 07 for a similar problem. What is the center of the power series for this problem?