Brief solutions to selected problems in homework 02

1. Section 8.8: Solutions, common mistakes and corrections:

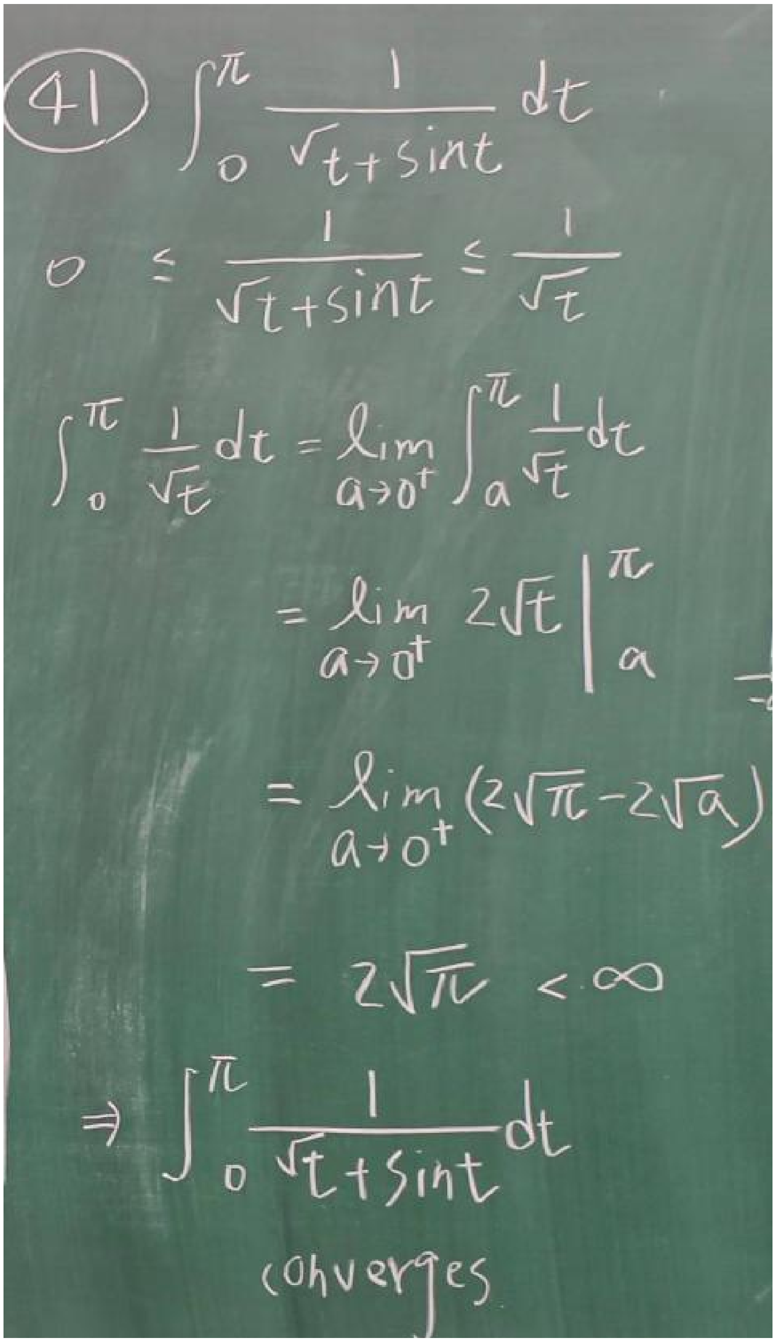

Figure 1: Solution to Section 8.8, problem 41

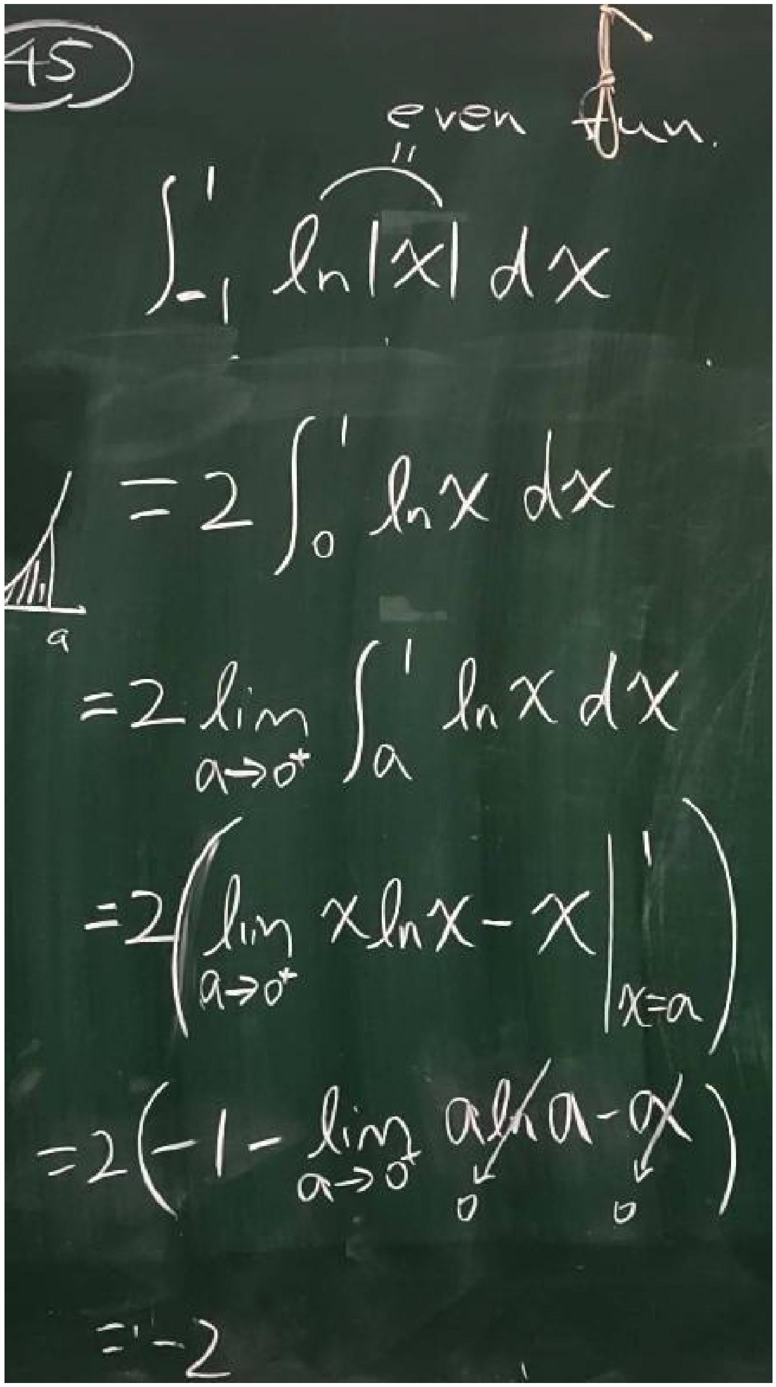

Figure 2: Solution to Section 8.8, problem 45

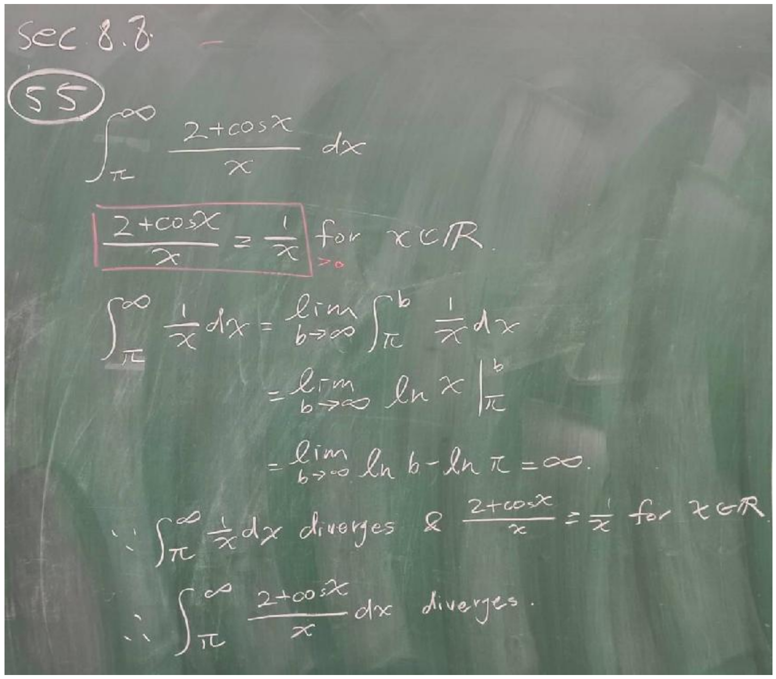

Figure 3: Solution to Section 8.8, problem 55

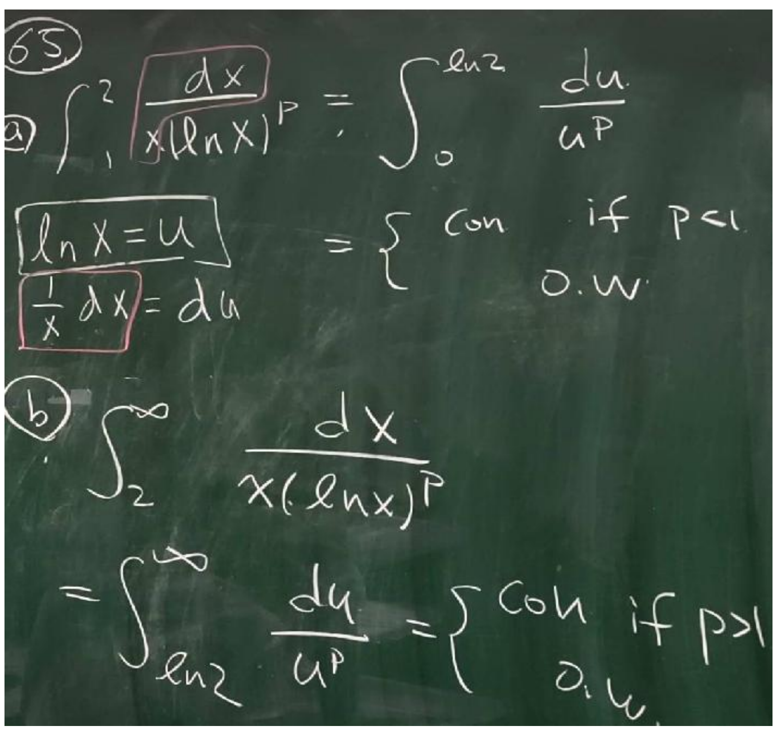

Figure 4: Solution to Section 8.8, problem 65

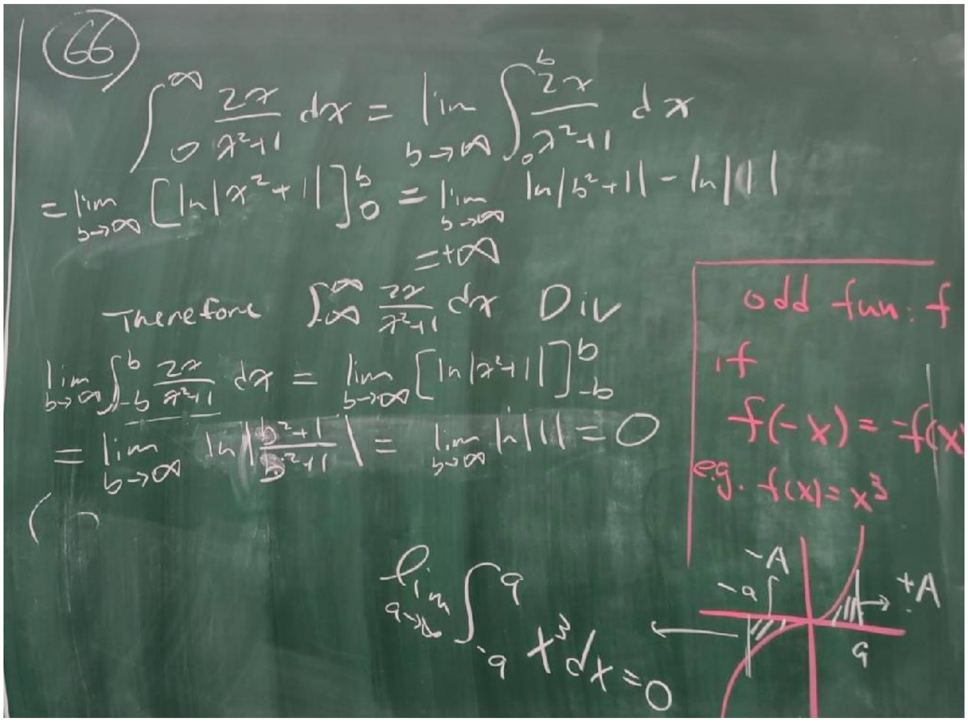

Figure 5: Solution to Section 8.8, problem 66

2. Section 8.8: Homework 02, problem 2:

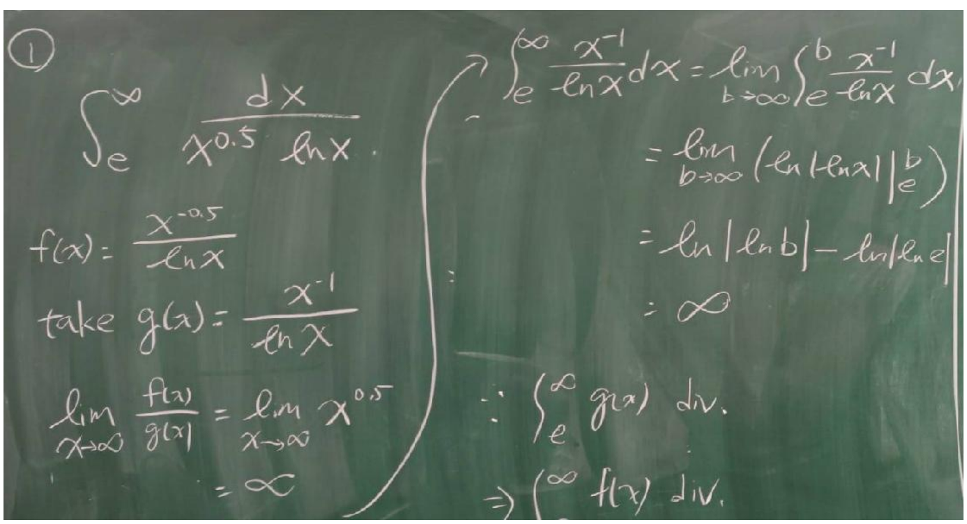

Figure 6: Solution to Section homework 02, problem 2

Remark: An alternative approach: Compare with $\int_e^\infty \frac{1}{x^{0.7}} dx$.

3. Section 8.8: Homework 02, problem 3:

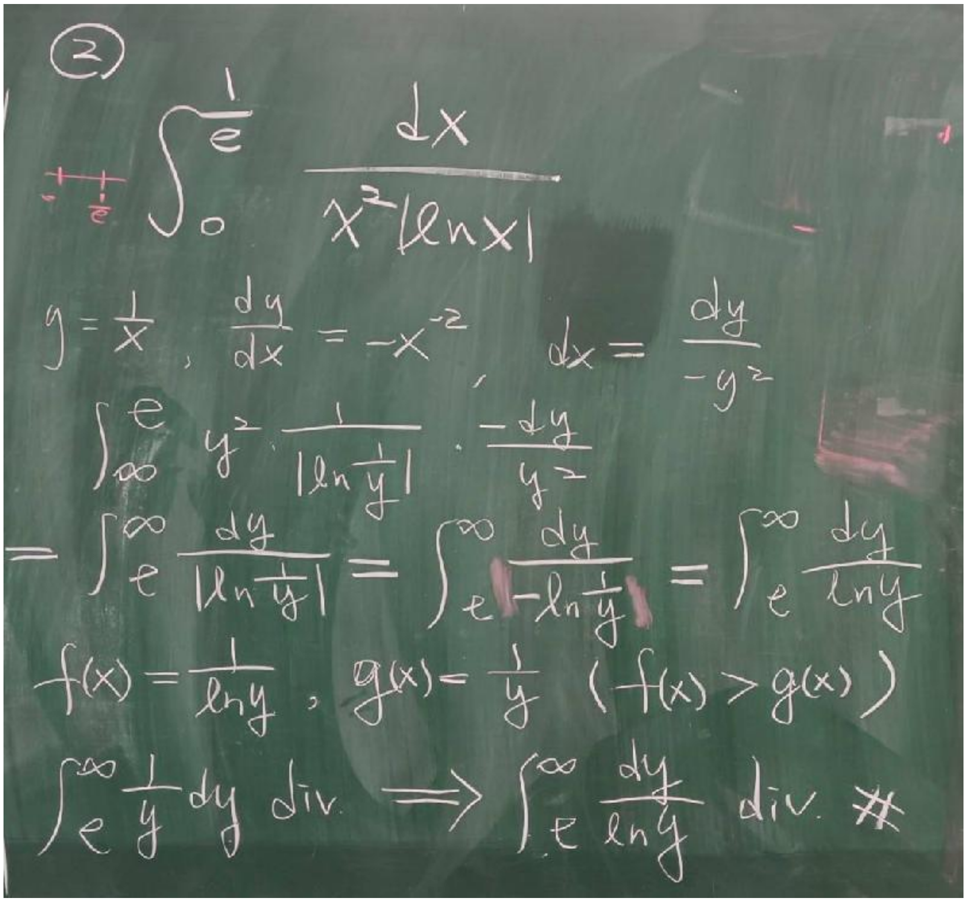

Figure 7: Solution to Section homework 02, problem 3

Remark: An alternative approach: Compare with $\int_0^{\frac{1}{e}} \frac{1}{x^{1.5}} dx$.

4. Section 10.1: Solutions, common mistakes and corrections:

   - Section 10.1, problem 46:

     Answer:

     Since

     $$0 \leq \frac{\sin^2 n}{2^n} \leq \frac{1}{2^n},$$

     and

     $$\lim_{n\to\infty} 0 = 0 = \lim_{n\to\infty} \frac{1}{2^n},$$

     we conclude from the Sandwich Theorem for sequence that

     $$\lim_{n\to\infty} \frac{\sin^2 n}{2^n} = 0.$$

- Section 10.1, problem 59:
  Answer:
  Since
  $$\lim_{n\to\infty} n^{\frac{1}{n}} = 1, \quad \lim_{n\to\infty} \ln n = \infty,$$
  we have
  $$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln n}{n^{\frac{1}{n}}} = \infty \text{ (diverges)}.$$

5. Section 10.2: Solutions, common mistakes and corrections:

- Section 10.2, problem 43:
  Answer:
  From the identity
  $$a_n = \frac{40n}{(2n-1)^2(2n+1)^2} = 5\Big(\frac{1}{(2n-1)^2} - \frac{1}{(2n+1)^2}\Big),$$
  we have
  $$s_k = \sum_{n=1}^{k} a_n = 5\Big(\frac{1}{(2\cdot 1-1)^2} - \frac{1}{(2\cdot 2-1)^2} + \frac{1}{(2\cdot 2-1)^2} - \cdots - \frac{1}{(2\cdot k+1)^2}\Big) = 5\Big(1 - \frac{1}{(2k+1)^2}\Big)$$
  Therefore
  $$\sum_{n=1}^{\infty} a_n = \lim_{k\to\infty} s_k = 5.$$
- Section 10.2, problem 61:
  Answer:
  Since $\lim_{n\to\infty} a_n = \infty \neq 0$, we conclude from the n-th term test that $\sum_{n=1}^{\infty} a_n$ diverges.
- Section 10.2, problem 65:
  Answer:
  From the identity
  $$a_n = \ln\left(\frac{n}{n+1}\right) = \ln n - \ln(n+1),$$
  we have
  $$s_k = \sum_{n=1}^{k} a_n = \ln 1 - \ln 2 + \ln 2 - \ln 3 + \cdots - \ln(k+1).$$
  Therefore
  $$\sum_{n=1}^{\infty} a_n = \lim_{k\to\infty} s_k = -\infty \text{ (diverges)}.$$

- Section 10.2, problem 78:
  Answer:

$$\text{The geometric series } \sum_{n=0}^{\infty} (\ln x)^n \text{ converges } \iff |\ln x| < 1 \iff \frac{1}{e} < x < e.$$

When $\frac{1}{e} < x < e$, $\displaystyle\sum_{n=0}^{\infty} (\ln x)^n = \frac{1}{1 - \ln x}$.