

Brief solutions to selected problems in homework 01

1. Section 8.8: Solutions, common mistakes and corrections:

$$\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2} dx \quad , \textcircled{0}$$
$$= \int_{-\infty}^0 \frac{2x}{(x^2+1)^2} dx + \int_0^{\infty} \frac{2x}{(x^2+1)^2} dx$$

(let $u=x^2+1$, $du=2x dx$, $x(-\infty, 0, \infty) \mapsto u(\infty, 1, \infty)$)

$$= \int_{\infty}^1 \frac{du}{u^2} + \int_1^{\infty} \frac{du}{u^2}$$
$$= \lim_{a \rightarrow \infty} \int_a^1 \frac{du}{u^2} + \lim_{b \rightarrow \infty} \int_1^b \frac{du}{u^2}$$
$$= \lim_{a \rightarrow \infty} \left. -\frac{1}{u} \right|_a^1 + \lim_{b \rightarrow \infty} \left. -\frac{1}{u} \right|_1^b$$
$$= -1 - \lim_{a \rightarrow \infty} \left(-\frac{1}{a}\right) + \lim_{b \rightarrow \infty} \left(-\frac{1}{b}\right) - (-1)$$
$$= -1 - 0 + 0 + 1 = 0$$

Figure 1: Solution to Section 8.8, problem 13

$$\int_0^1 x \ln x \, dx, \quad \textcircled{-\frac{1}{4}}$$

$$\lim_{b \rightarrow 0^+} \int_b^1 \underbrace{x \ln x}_{u \cdot v} \, dx \quad \begin{array}{l} u'v = \\ - \int u \cdot v' \end{array}$$

$$= \lim_{b \rightarrow 0^+} \left(\ln x \cdot \left[\frac{1}{2} x^2 \right] - \int_b^1 \frac{1}{2} x^2 \cdot \frac{1}{x} \, dx \right)$$

$$= \lim_{b \rightarrow 0^+} \left(\frac{1}{2} \ln x \cdot x^2 - \frac{1}{4} x^2 \Big|_b^1 \right)$$

$$= \lim_{b \rightarrow 0^+} \left(0 - \left[\frac{1}{2} \ln b \cdot b^2 \right] - \left(\frac{1}{4} - \frac{1}{4} b^2 \right) \right)$$

$$= -\frac{1}{4} \#$$

Figure 2: Solution to Section 8.8, problem 25

$$\int_{-1}^4 \frac{dx}{\sqrt{|x|}}, \quad \textcircled{6}$$

$$\lim_{a \rightarrow 0^-} \int_{-1}^a (-x)^{-\frac{1}{2}} \, dx = \underbrace{-2(-x)^{\frac{1}{2}} \Big|_{-1}^a}_{\lim_{a \rightarrow 0^-} \frac{1}{4}} = \underbrace{2\sqrt{a} - 2}_{\lim_{a \rightarrow 0^-} 2} = 2$$

$$\lim_{b \rightarrow 0^+} \int_b^4 x^{-\frac{1}{2}} = \underbrace{2x^{\frac{1}{2}} \Big|_b^4}_{\lim_{b \rightarrow 0^+} 2} = 4 + 0 = 4$$

$$2 + 4 = 6$$

Figure 3: Solution to Section 8.8, problem 31