

## Homework 15

1. Section 16.1: Problems 15, 23, 25, 29.

2. Section 16.2: Problems 19, 25, 29(b), 35(a).

See equation (5) in p978 and (6) in p979 for definition of flow, circulation and flux.

3. Section 16.3: Problems 1, 3, 5, 9, 21, 26, 29.

4. Section 16.3:

Let  $\mathbf{F} = \frac{x}{\sqrt{x^2 + y^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}} \mathbf{j} + 0 \mathbf{k}$  and  $\mathbf{G} = \frac{-y}{x^2 + y^2} \mathbf{i} + \frac{x}{x^2 + y^2} \mathbf{j} + 0 \mathbf{k}$ .

(a) Show that both  $\mathbf{F}$  and  $\mathbf{G}$  satisfy the component test.

(b) The natural domain of both  $\mathbf{F}$  and  $\mathbf{G}$  is  $\{(x, y, z), x^2 + y^2 \neq 0\}$  (that is where  $\mathbf{F}$  and  $\mathbf{G}$  are defined). Show that  $\mathbf{F}$  is conservative in this domain by finding its potential function.

(c) Show that  $\mathbf{G}$  is NOT conservative in this domain (see Example 5 on p990).

(d) If given another  $\mathbf{H}$  satisfying the component test in this domain, how do you determine whether  $\mathbf{H}$  is conservative?

5. Section 16.3:

Let  $\mathbf{F} = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2 + z^2}} \mathbf{j} + \frac{z}{\sqrt{x^2 + y^2 + z^2}} \mathbf{k}$ . What is the natural domain of  $\mathbf{F}$ ? Show that  $\mathbf{F}$  satisfies the component test in this domain. Is this domain simply connected? Is  $\mathbf{F}$  conservative in this domain?