# Homework 14

1. Section 15.7: Problems 9, 11, 13, 14, 15, 19, 21, 31, 39, 43, 45.

2. Section 15.8: Problems 5, 7, 9, 15, 19.

3. Section 15.8: Compute the Jacobians for the substitutions $x = r\cos\theta$, $y = r\sin\theta$ in double integral, $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ and $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$ $z = \rho\cos\phi$ in triple integral, respectively.

4. Chap 15, Additional and Advanced Exercises: Problems 11, 12.

5. (Optional)

   Section 14.10: Follow up on problem 3 of homework 12:

   Evaluate the constrained second derivatives of $w$ at the points $(x_0, y_0, z_0)$ obtained in homework 12, problem 3. That is, evaluate $\left(\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right)_y\right)_y$, $\left(\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right)_x\right)_x$ and $\left(\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right)_y\right)_x = \left(\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right)_x\right)_y$ at $(x_0, y_0, z_0)$ and use them to determine whether $(x_0, y_0, z_0)$ is a local min, local max, or saddle point of $w$ on the constraint set $x^2 + y^2 + z^2 = 30$.

   Hint: First evaluate $\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)_y\right)_y$, $\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)_x\right)_x$ and $\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)_y\right)_x = \left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)_x\right)_y$ at $(x_0, y_0, z_0)$ from the constraint $x^2 + y^2 + z^2 = 30$.

   Remark: Both methods (Lagrange Multiplier and Partial Derivative with Constrained Variables) can be applied to find easily the critical points of $w$ on the constraint set. However, the second derivative test for Lagrange Multiplier is much more complicated (try google it and you will find it difficult even to understand the statement). In contrast, the second derivative test for Partial Derivative with Constrained Variables is straight forwardly as outline above.