Homework 12

1. Section 14.10: Problems 3, 7, 9, 12.

   Hint for Problem 12: Lecture 23, page 4.

2. Section 14.10: Follow up on problem 12:

   Derive a formula for $\left(\frac{\partial u}{\partial x}\right)_y$ if $u = U(x, y, z, w)$, $f(x, y, z, w) = 0$ and $g(x, y, z, w) = 0$ provided $f_z g_w - f_w g_z \neq 0$.

   Hint: count the numbers of dependent and independent variables first. Then use the result in Problem 12 to find $\left(\frac{\partial w}{\partial x}\right)_y$ and $\left(\frac{\partial z}{\partial x}\right)_y$ first, and then $\left(\frac{\partial u}{\partial x}\right)_y$.

   You can first take a specific example such as $U(x, y, z, w) = x^2 + y^2 + z^2 + w^2$, $f(x, y, z, w) = x + y + z^2 + w^2$, $g(x, y, z, w) = x^2 + y^2 + z + w$. Then generalize the result to general U, f and g.

3. Section 14.10: Compute the partial derivatives with constrained variables $\left(\frac{\partial w}{\partial x}\right)_y$ and $\left(\frac{\partial w}{\partial y}\right)_x$ if $w = x-2y+5z$ and $x^2+y^2+z^2 = 30$. Then find (x_0, y_0, z_0) on $x^2+y^2+z^2 = 30$ such that

$$\left(\frac{\partial w}{\partial x}\right)_y (x_0, y_0, z_0) = 0, \quad \left(\frac{\partial w}{\partial y}\right)_x (x_0, y_0, z_0) = 0. \tag{1}$$

   Remark: This is an alternative method for Section 14.8, problem 23. You can also change (1) to

$$\left(\frac{\partial w}{\partial y}\right)_z (x_0, y_0, z_0) = 0, \quad \left(\frac{\partial w}{\partial z}\right)_y (x_0, y_0, z_0) = 0, \tag{2}$$

   or

$$\left(\frac{\partial w}{\partial z}\right)_x (x_0, y_0, z_0) = 0, \quad \left(\frac{\partial w}{\partial x}\right)_z (x_0, y_0, z_0) = 0 \tag{3}$$

   and still get the same solution (x_0, y_0, z_0) (why?).

4. Section 15.1: Problems 21, 33, 36.

5. Section 15.2: Problems 11, 19, 29, 35, 43, 47, 69.

   Remark on problem 47: A special trick will be explained in Lecture 25, 240523.