Homework 09

1. Section 14.4: Problems 1, 7, 10, 21, 24, 29, 31, 43, 51.
2. Section 14.5: Problems 9, 15, 19, 25, 27, 29, 35, 36, 40 (See page 850).
3. Section 14.5: Let $f_1(x,y) = \sqrt{x^2+y^2}^{\frac{1}{2}} = (x^2+y^2)^{\frac{1}{4}}$,
   $f_2(x,y) = 2x+3y+4+\sqrt{x^2+y^2}^{\frac{3}{2}} = 2x+3y+4+(x^2+y^2)^{\frac{3}{4}}$ and
   $f_3(x,y) = \dfrac{x^3}{x^2+y^2}$ for $(x,y) \neq (0,0)$, $f_3(0,0) = 0$.
   (a) Are f_i continuous at $(0,0)$?
   (b) Do $\partial_x f_i$ and $\partial_y f_i$ exist at $(0,0)$?
   (c) Use the definition of directional derivative to evaluate $\dfrac{df_i}{ds}_{(0,0),(\cos\theta,\sin\theta)}$, i.e. the directional derivative of f_i at $(x_0,y_0) = (0,0)$ in the direction $(\cos\theta,\sin\theta)$, if it exists.
   (d) Are f_i differentiable at $(0,0)$?
   Hint:
      i. If you know or guess that f is differentiable at (x_0,y_0), you can try to prove it using Theorem 3 (Section 14.3, page 832). It may or may not work.
      ii. If you know or guess that f is NOT differentiable at (x_0,y_0), you can try to prove it using Theorem 4 (Section 14.3, page 832) or Theorem 9 (Section 14.5, page 847). It may or may not work.
      iii. Note: You definitely CANNOT use Theorem 3 in (ii) or Theorem 4, Theorem 9 in (i). If you don't know whether f is differentiable at (x_0,y_0) or not, it is always safe to follow the definition as outlined in page 7 of Lecture 15. It almost always works.