Homework 05

1. Section 10.7: 51, 55, 57, 60.
2. Section 10.7: Use the power series representation of $\dfrac{1}{1 \pm x}$ to find the power series representation of $\ln(1 \pm x)$ on $|x| < 1$.
3. Section 10.7: Find the first three nonzero terms of the power series representation of
$$\frac{1 - x^2 + x^4 - \cdots}{1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \cdots}$$
by "long division".
4. Section 10.8: Problems 5 ($n = 3$), 7 ($n = 3$), 15, 23, 29, 35.

   Hint for problem 5:

   Method 1: Find $f^{(n)}(a)$ by repeated differentiation.

   Method 2: Write $\dfrac{1}{x} = \dfrac{1}{x - 2 + 2} = \dfrac{1}{2} \dfrac{1}{\left(1 + \dfrac{x-2}{2}\right)}$ and use the formula for geometric series to find the power series representation of $f(x)$. This will give you Taylor series generated by f at a (Theorem A, Lecture 09) and therefore Taylor polynomial generated by f at a.

   Hint for problem 35:

   Method 1: Find $f^{(n)}(a)$ by repeated differentiation.

   Method 2 (easier): Use problem 2 above, Theorem 19 of section 10.7 and finally Theorem A, Lecture 09.

   Update: also need to use a result from Section 10.9, Example 2: $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots$ valid for all $x \in \mathbb{R}$ (that is, radius of convergence $R > 0$).

Theorem A (See page 10 of Lecture 09):
If $f(x)$ has a power series representation on $|x - a| < R$ with $\underline{R > 0}$, then $T_{f,a}(x) = f(x)$.

In other words:

If $f(x) = \sum_{k=0}^{\infty} a_k (x-a)^k$ on $|x - a| < R$ with $\underline{R > 0}$, then $T_{f,a}(x) = \sum_{k=0}^{\infty} a_k (x-a)^k$.