Homework 03

1. Section 10.3: Problems 7, 27, 28, 31, 33, 37, 41, 55.

   Hint for problem 41: If both $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n \pm b_n)$ could either converge or diverge in general.

   To determine the convergence or divergence, one can, for example, simplify $a_n = \frac{a}{n+2} - \frac{1}{n+4} = \frac{\cdots}{(n+2)(n+4)}$ and use one of the comparison methods in Section 10.4.

2. Section 10.4: Problems 15, 16, 17, 27, 29, 31, 43, 45, 51, 61, 62.

   Hints for problem 61:

   (a) For simplicity, just take a fixed case $p = 1.5$, $q = 3$ and proceed. The same argument works for all $p > 1$, $q > 0$. The case $p > 1$, $q \leq 0$ is easier (Why?).

   (b) Instead of evaluating $\lim_{n\to\infty} \frac{(\ln n)^3}{n^{1.5-1.25}}$, it is and easier to evaluate $\left(\lim_{n\to\infty} \frac{\ln n}{n^{\frac{1.5-1.25}{3}}}\right)^3$.

   Hint for problem 62:

   For simplicity, just take a fixed case $p = 0.5$, $q = -2$ and proceed. The same argument works for all $0 < p < 1$, $q < 0$. The case $0 < p < 1$, $q \geq 0$ is easier.

3. Section 10.5: Odd numbered problems in problem 17-43, 61, 65.

   Hint: In general, the ratio test applies if you see the factorial $(\cdots)!$ appearing in a_n. We did not have time for more examples in lecture 04. Do your best as time permits. We will proceed with more examples (and the proof) for both tests in lecture 05.

   Hint for problem 61: multiply by $2 * 4 * \cdots * 2n = 2^n n!$ both on the denominator and the numerator.

   Hint for problem 65: Ratio test will be inconclusive, but root test works.