Calculus II, Spring 2023 (http://www.math.nthu.edu.tw/~wangwc/)

Thomas' Calculus Early Transcendentals 13ed

Study guide for quiz 08

Quiz problems include both the lecture contents and homework problems.

1. Section 15.1, 15.2: Study how to identify the limits of integration in $\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dxdy$ and $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dydx$ for general domains (that is, not rectangles).

2. Section 15.2, 15.3: Study how to interchange between $\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dxdy$ and $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dydx$ for general domains as in problems 33-56 of section 15.2.

3. Section 15.4:

   Study why $dA = rdrd\theta$ in polar coordinates. Practice how to determine the limits of integration in $\int_{\theta_1}^{\theta_2} \int_{f_1(\theta)}^{f_2(\theta)} (\cdots) rdrd\theta$ as in Examples 2-6 of section 15.4. More specifically:

   (a) Given a domain R in the $x-y$ plane, practice drawing $\theta = C$ lines in R. The end points of these lines are lower limit (the near end point) and upper limit (the far end point) of the integration rdr. The end points for $d\theta$ are smallest and largest C among these $\theta = C$ lines.

   (b) The end points of the lines $\theta = C$ must be expressed as $r = f_1(\theta)$ and $r = f_2(\theta)$. Given a simple curve $F(x,y) = 0$ (such as a line or a circle), use the substitution $x = r\cos\theta$, $y = r\cos\theta$ to express it as $r = f(\theta)$. Examples: $x = 1$, $y = -3$, $x + y = 1$, $x^2 + y^2 = 4$, etc.