Study guide for Final Exam

Final Exam problems include both the lecture contents and homework problems.

1. Review study guide for quiz 08-10.

2. Section 16.3:

   Study and memorize the definitions of 'path independent', 'conservative' and 'potential function' (p984).

   Study the statement and proof of Theorem 1:'Fundamental Theorem of Line Integrals' (p985).

   Study the definition and examples (Figure 16.22) of 'connected domain' and 'simply connected domain' (p985).

   Study the relation between conservative fields and

   (a) 'Gradient Fields' (Theorem 2, p986).
   (b) 'Loop Property' (Theorem 3, p987)
   (c) 'Component test' (p988),

   Which of them are equivalent to 'Conservative Fields'?

   Which of them are equivalent to 'Conservative Fields' only on simply connected domains?

   Which of the implications '$\Longleftarrow$', '$\Longrightarrow$' remains valid even if the domain is not simply connected (review homework 13, problem 2 for counter examples)?

   For given functions $M(x,y,z), N(x,y,z), P(x,y,z)$ satisfying the component test, how does one find the potential function (if it exists) by way of direct integration (Example 3)?

3. Section 16.3:

   Skip "Exact Differential Forms" on page 991-992.

4. Section 16.4:

   Study and memorize Green's Theorem both in tangential form and normal form (p1000).

   Is Theorem $\underline{4}$ applicable to Example $\underline{5}$ of Section 16.3? Which part went wrong?

5. Section 16.4:

   The double integral in Green's Theorem involves

$$\text{div}\boldsymbol{F}(x,y) = M_x(x,y) + N_y(x,y) \tag{1}$$

   and

$$\text{curl}\boldsymbol{F}(x,y)\cdot\boldsymbol{k} = N_x(x,y) - M_y(x,y) \tag{2}$$

It may be useful to help you memorize them if you start with

$$\text{div}\boldsymbol{F}(x, y, z) = \nabla \cdot \boldsymbol{F}(x, y, z) = (\partial_x, \partial_y, \partial_z) \cdot \boldsymbol{F}(x, y, z) = M_x(x, y, z) + N_y(x, y, z) + P_z(x, y, z)$$

and

$$\text{curl}\boldsymbol{F}(x, y, z) = \nabla \times \boldsymbol{F}(x, y, z) = (\partial_x, \partial_y, \partial_z) \times \boldsymbol{F}(x, y, z) = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \partial_x & \partial_y & \partial_z \\ M & N & P \end{vmatrix}$$

and view (1) and (2) as special case of $\boldsymbol{F} = (M(x, y),\ N(x, y),\ 0)$.