Brief solutions to Quiz 9

May 30, 2023:

1. (34 pts) Let D be the domain enclosed by $x = 0$, $z = 0$, $y = 1$ and $x - y + z = 0$ as shown in the figure. Express the volume of D in terms of $dxdydz$ (in this order only). The answer is $\dfrac{1}{6}$. You can use it to check if you have the correct expression.

   Ans:
   $$V = \int_{z=0}^{1} \int_{y=z}^{1} \int_{x=0}^{y-z} dxdydz$$

2. (66 pts) Let D be the region bounded below by $z = 0$, above by $x^2 + y^2 + z^2 = 4$ and the sides by $x^2 + y^2 = 1$. Express the volume of D in terms of $dzdrd\theta$ and $d\rho d\phi d\theta$, respectively. The answer is $\left(\dfrac{8}{3} - \sqrt{3}\right) \cdot 2\pi$. You can use it to check if you have the correct expressions.

   Ans:
   $$V = \int_{\theta=0}^{2\pi} \int_{r=0}^{1} \int_{z=0}^{\sqrt{4-z^2}} dz\ r dr\ d\theta$$
   $$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\frac{\pi}{6}} \int_{\rho=0}^{2} \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta + \int_{\theta=0}^{2\pi} \int_{\phi=\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{\rho=0}^{\csc\phi} \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$$
   $$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\frac{\pi}{2}} \int_{\rho=0}^{2} \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta - \int_{\theta=0}^{2\pi} \int_{\phi=\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{\rho=\csc\phi}^{2} \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$$