Brief solutions to Quiz 5

Apr 18, 2023:

1. (40 pts) (**Average = 27.42 pts**)

   Give formal definition of $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$. Does $f(x,y) = \dfrac{xy^2-1}{y-1}$ have a limit at $(1,1)$? Explain.

   Ans:

   Definition: See page 816 of the textbook.

   No. See "Homework 06 solution" (Section 14.2, problem 49) from Week 07 of the course homepage.

2. (40 pts) (**Average = 12.58 + 11.49 pts**)

   (a) Give formal definition of '$f(x,y)$ is differentiable at (x_0,y_0)'.

   (b) True or False? Explain:
   If $f(x,y)$ is differentiable at (x_0,y_0), then it is continuous at (x_0,y_0).

   Ans:

   (a): See page 832 of the textbook, or the link 'Remark on definition of differentiability of $f(x,y)$" from Week 09 of the course homepage. Any one of the equivalent definitions will do.

   (b): True. See page 832 of the textbook, or page 7 of Lecture 14.

3. (20 pts) (**Average = 16.94 pts**)

   Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(x,y,z) = (1,1,1)$ if $z(x,y)$ is implicitly defined by $xy + z^3x - 2yz = 0$.

   Ans:
   $\dfrac{\partial z}{\partial x}(1,1,1) = -2$, $\dfrac{\partial z}{\partial y}(1,1,1) = 1$. See "Homework 06 solution" (Section 14.3, problem 65) from Week 07 of the course homepage for details.