Brief solutions to Quiz 3

Mar 14, 2023:
Average = 80.14 pts

1. (20+20+20 pts. Correct answer: 8 pts. Correct explanation: 12 pts) (**Average = 10.90 pts + 18.63 pts + 17.38 pts**)

   Are the following series convergent? Explain.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^3}$

   Ans: Convergent. Since it converges absolutely (by direct comparison with the p-series, $p = 3$). From Absolute Convergence Test, this series converges.

   Remark:

   Many of you have used the "Sandwich Theorem for Series" (Note: completely different from "Sandwich Theorem for Sequences"). Some points were deducted for taking this approach without giving the complete statement of this Theorem, since this Theorem, although correct, was not mentioned in class nor in the textbook.

   For your reference, this Theorem can be found in this document together with a simple proof at the bottom of this document (you can skip everything in between). It is worth noting that the proof is basically identical to the proof of "Absolute Convergence Test" (absolute convergence implies convergence) that we used above.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$

   Ans: Convergent by ratio test ($\rho = \dfrac{1}{3}$). See also homework solution for section 10.5 problem 35.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$

   Ans: Convergent by root test ($\rho = \dfrac{1}{10}$). See also homework solution for section 10.5 problem 21.

2. (Correct statement: 20 pts. Correct example: 8 pts. Correct explanation: 12 pts) (**Average = 17.10 pts + 16.14**)

   State the Alternating Series Test (need not prove it) and give an example of a conditionally convergent series (give details).

   Ans:

Statement: See Theorem 15, Section 10.6.

Example: $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}$. It is convergent by the Alternating Series Test. It does not converge absolutely (by direct comparison with the p-series, $p = 1$).