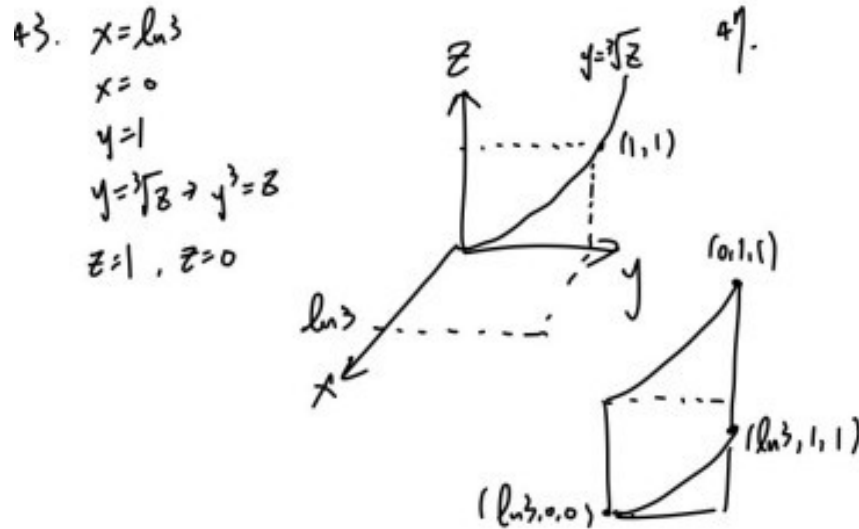


Brief solutions to selected problems in homework 11

1. Section 15.5: Solutions, common mistakes and corrections:



$$\int_0^1 \int_0^{\ln 3} \int_0^{y^3} \frac{z e^{zx} \sin zy^2}{y^2} dz dx dy$$

$$= \int_0^1 \int_0^{\ln 3} z e^{zx} \sin(zy^2) y dx dy$$

$$= \int_0^1 \frac{z}{y} \sin(zy^2) e^{zx} \Big|_0^{\ln 3} dy$$

$$= \int_0^1 \frac{z}{y} \sin(zy^2) (9 - 1) dy$$

$$= \frac{8}{2} \int_0^1 y \sin(zy^2) dy = \frac{-8z}{2} \frac{1}{2z} \cos(zy^2) \Big|_0^1$$

$$= -2 (\cos \pi - \cos 0) = 4$$

Figure 1: Section 15.5, problem 43

2. Section 15.7: Solutions, common mistakes and corrections:

Section 15.7, problem 14:

Ans:

$$\int_{z=0}^x dz = \int_{z=0}^{r \cos \theta} dz, \quad \int_{y=-1}^1 \int_{x=0}^{\sqrt{1-y^2}} dx dy = \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^1 r dr d\theta$$

$$I = \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^1 \int_{z=0}^{r \cos \theta} r^2 dz r dr d\theta = \int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{r=0}^1 r^2 \cdot (r \cos \theta) r dr d\theta = \frac{2}{5}$$

Section 15.7, problem 31:

31. Let  $D$  be the region in Exercise 11. Set up the triple integrals in spherical coordinates that give the volume of  $D$  using the following orders of integration.
- a.  $d\rho d\phi d\theta$       b.  $d\phi d\rho d\theta$
11. Let  $D$  be the region bounded below by the plane  $z = 0$ , above by the sphere  $x^2 + y^2 + z^2 = 4$ , and on the sides by the cylinder  $x^2 + y^2 = 1$ . Set up the triple integrals in cylindrical coordinates that give the volume of  $D$  using the following orders of integration.
- a.  $dz dr d\theta$       b.  $dr dz d\theta$       c.  $d\theta dz dr$

a.  $x^2 + y^2 = 1$

$$\Rightarrow \rho^2 \sin^2 \phi = 1$$

$$\rho \sin \phi = 1$$

$$\rho = \csc \phi$$

$$\Rightarrow \int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^{\csc \phi} \rho^2 \sin \phi d\rho d\phi d\theta$$

$$+ \int_0^{2\pi} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_0^{\csc \phi} \rho^2 \sin \phi d\rho d\phi d\theta$$

b.  $\int_0^{2\pi} \int_1^2 \int_{\sin^{-1}(\frac{1}{\rho})}^{\frac{\pi}{6}} \rho^2 \sin \phi d\phi d\rho d\theta$

$$+ \int_0^{2\pi} \int_0^2 \int_0^{\frac{\pi}{6}} \rho^2 \sin \phi d\phi d\rho d\theta$$

$$+ \int_0^{2\pi} \int_0^1 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \rho^2 \sin \phi d\phi d\rho d\theta$$

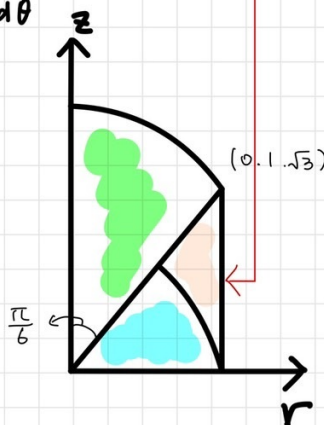


Figure 2: Section 15.7, problem 31

Other correct answers for part (b):

$$\begin{aligned} &= \int_{\theta=0}^{2\pi} \int_{\rho=1}^2 \int_{\phi=0}^{\sin^{-1}(\frac{1}{\rho})} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta + \int_{\theta=0}^{2\pi} \int_{\rho=0}^1 \int_{\phi=0}^{\frac{\pi}{2}} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta \\ &= \int_{\theta=0}^{2\pi} \int_{\rho=0}^2 \int_{\phi=0}^{\frac{\pi}{2}} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta - \int_{\theta=0}^{2\pi} \int_{\rho=1}^2 \int_{\phi=\sin^{-1}(\frac{1}{\rho})}^{\frac{\pi}{2}} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta \end{aligned}$$