Brief solutions to selected problems in homework 11

1. Section 15.5: Solutions, common mistakes and corrections:

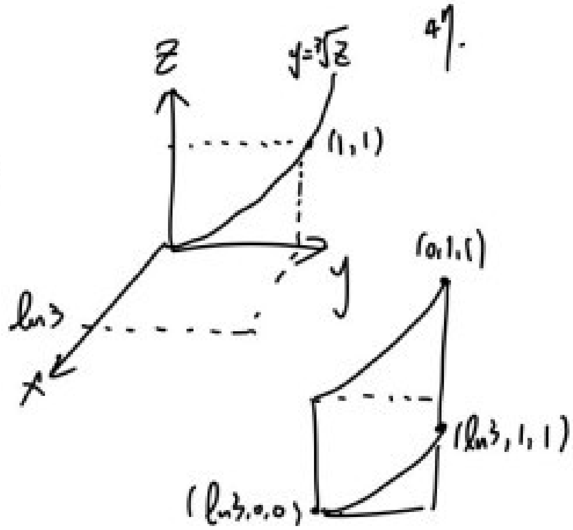
43. $x = \ln 3$

$x = 0$

$y = 1$

$y = \sqrt[3]{z} \rightarrow y^3 = z$

$z = 1,\ z = 0$



$$\int_0^1 \int_0^{\ln 3} \int_0^{y^3} \frac{\pi e^{2x} \sin \pi y^2}{y^2}\, dz\, dx\, dy$$

$$= \int_0^1 \int_0^{\ln 3} \pi e^{2x} \sin(\pi y^2) y \, dx\, dy$$

$$= \int_0^1 \frac{\pi}{2} y \sin(\pi y^2) e^{2x} \Big|_0^{\ln 3} dy$$

$$= \int_0^1 \frac{\pi}{2} y \sin(\pi y^2)(9-1)\, dy$$

$$= \frac{8}{2}\pi \int_0^1 y \sin(\pi y^2)\, dy = \frac{-8\pi}{2} \frac{1}{2\pi} \cos(\pi y^2)\Big|_0^1$$

$$= -2(\cos \pi - \cos 0) = 4$$

Figure 1: Section 15.5, problem 43

2. Section 15.7: Solutions, common mistakes and corrections:

Section 15.7, problem 14:

Ans:

$$\int_{z=0}^{x} dz = \int_{z=0}^{r\cos\theta} dz, \quad \int_{y=-1}^{1}\int_{x=0}^{\sqrt{1-y^2}} dx\, dy = \int_{\theta=\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_{r=0}^{1} r\, dr\, d\theta$$

$$I = \int_{\theta=\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_{r=0}^{1}\int_{z=0}^{r\cos\theta} r^2\, dz\, rdr\, d\theta = \int_{\theta=\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_{r=0}^{1} r^2\cdot(r\cos\theta)\, rdr\, d\theta = \frac{2}{5}$$

Section 15.7, problem 31:

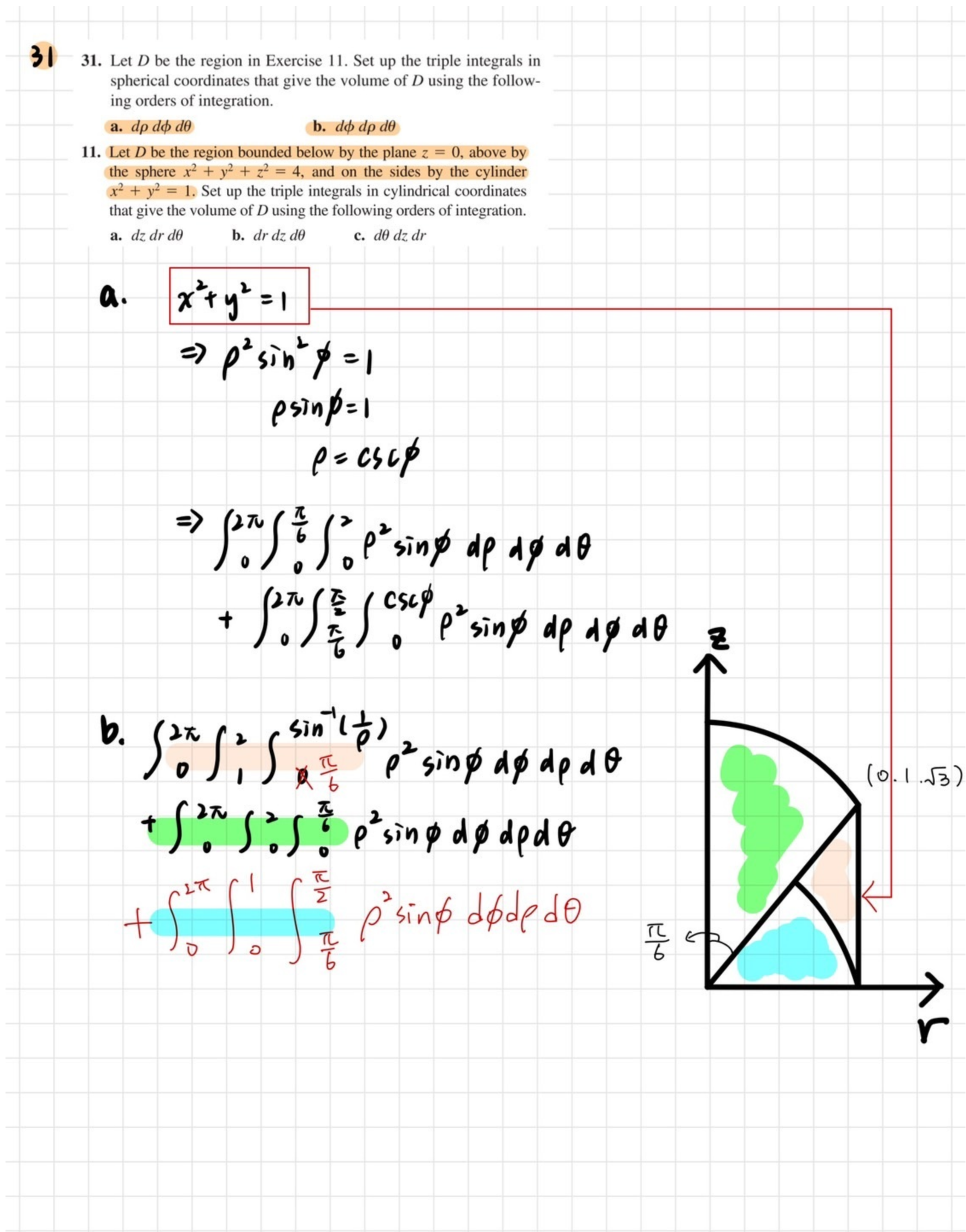


Figure 2: Section 15.7, problem 31

Other correct answers for part (b):

$$= \int_{\theta=0}^{2\pi} \int_{\rho=1}^{2} \int_{\phi=0}^{\sin^{-1}(\frac{1}{\rho})} \rho^2 \sin\phi \; d\phi \; d\rho \; d\theta + \int_{\theta=0}^{2\pi} \int_{\rho=0}^{1} \int_{\phi=0}^{\frac{\pi}{2}} \rho^2 \sin\phi \; d\phi \; d\rho \; d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{\rho=0}^{2} \int_{\phi=0}^{\frac{\pi}{2}} \rho^2 \sin\phi \; d\phi \; d\rho \; d\theta - \int_{\theta=0}^{2\pi} \int_{\rho=1}^{2} \int_{\phi=\sin^{-1}(\frac{1}{\rho})}^{\frac{\pi}{2}} \rho^2 \sin\phi \; d\phi \; d\rho \; d\theta$$