

Brief solutions to selected problems in homework 07

1. Section 14.4: Solutions, common mistakes and corrections:

10. a) $\frac{\partial w}{\partial u} = \left(\frac{2x}{x^2+y^2+z^2}\right)(e^v \sin u + u e^v \cos u)$ 21.

Method 1

$$+ \left(\frac{2y}{x^2+y^2+z^2}\right)(e^v \cos u - u e^v \sin u) + \left(\frac{2z}{x^2+y^2+z^2}\right)(e^v)$$

$$= \left(\frac{2u e^v \sin u}{u^2 e^{2v} \sin^2 u + u^2 e^{2v} \cos^2 u + e^{2v}}\right)(e^v \sin u + u e^v \cos u)$$

$$+ \left(\frac{2u e^v \cos u}{u^2 e^{2v} \sin^2 u + u^2 e^{2v} \cos^2 u + e^{2v}}\right)(e^v \cos u - u e^v \sin u)$$

$$+ \left(\frac{2u e^v}{u^2 e^{2v} \sin^2 u + u^2 e^{2v} \cos^2 u + e^{2v}}\right)(e^v) = \frac{2}{u}$$

Method 2

$$w = \ln(x^2 + y^2 + z^2) = \ln(2u^2 e^{2v})$$

$$= \ln 2 + 2 \ln u + 2v$$

$$\frac{\partial w}{\partial u} = \frac{2}{u}, \quad \frac{\partial w}{\partial v} = 2$$

Figure 1: Section 14.4, problem 10

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial s} = g'(h(s,t)) \partial_s h(s,t)$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial t} = g'(h(s,t)) \partial_t h(s,t)$$

$$w = g(u), \quad u = h(s,t)$$

Figure 2: Section 14.4, problem 21

$$\begin{aligned}
 24. \quad \frac{\partial w}{\partial s} \text{ for } w = g(x, y) \\
 \begin{aligned}
 x &= h(r, s, t) \\
 y &= k(r, s, t)
 \end{aligned} \\
 \frac{\partial w}{\partial s} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} \\
 &= \frac{\partial g}{\partial x}(h(r, s, t), k(r, s, t)) \cdot \frac{\partial h}{\partial s}(r, s, t) \\
 &\quad + \frac{\partial g}{\partial y}(h(r, s, t), k(r, s, t)) \cdot \frac{\partial k}{\partial s}(r, s, t)
 \end{aligned}$$

Figure 3: Section 14.4, problem 24

14.4 p51

$$(i) F(x) = \int_0^{x^2} \sqrt{t^4 + t^3} dt$$

$$(ii) G(u, x) = \int_0^u \sqrt{t^4 + t^3} dt$$

$$\Rightarrow F'(x) = \frac{\partial G}{\partial u} \frac{du}{dx} + \frac{\partial G}{\partial x} \cdot \frac{dx}{dx}$$

$$(iv) = \sqrt{x^8 + x^3} \cdot 2x + \int_0^{x^2} \frac{\partial}{\partial x} \sqrt{t^4 + t^3} dt$$
$$= 2x \sqrt{x^8 + x^3} + \int_0^{x^2} \frac{3x^2}{2\sqrt{t^4 + t^3}} dt$$

$$(iii) F(x) = G(u(x), x)$$

$$\text{with } u(x) = x^2$$

$$(iv) F'(x) = \partial_1 G \cdot u'(x) + \partial_2 G \cdot 1$$
$$= \sqrt{u^4 + x^3} \cdot u'(x) + \int_0^u \frac{\partial}{\partial x} \sqrt{t^4 + t^3} dt \Big|_{u=x^2}$$

$$(iii') F(x) = G(u, x), (u = x^2)$$

$$F'(x) = \partial_u G \cdot \partial_x u + \partial_x G \cdot 1$$

= the same

Figure 4: Section 14.4, problem 51

2. Section 14.5: Solutions, common mistakes and corrections:

14.5.24
 $f(x,y) = x^2 - xy + y^2 - y$

(a) $\nabla f(1,-1) = 3i - 4j$ $|\nabla f(1,-1)| = 5$, $D_{\vec{u}}f(1,-1) = 5 \Rightarrow \vec{u} = \frac{3}{5}i - \frac{4}{5}j$
 $\left(= \frac{\nabla f}{|\nabla f|} \right)$

(b) $-\nabla f(1,-1) = -3i - 4j$; $\vec{u} = -\frac{3}{5}i + \frac{4}{5}j$
 $\left(= -\frac{\nabla f}{|\nabla f|} \right)$

(c) $D_{\vec{u}}f(1,-1) = 0$, $\vec{u} = \frac{4}{5}i + \frac{3}{5}j$ or $-\frac{4}{5}i - \frac{3}{5}j$
 $(\text{e.g. } \vec{u} \perp \nabla f)$

(d) Let $\vec{u} = u_1i + u_2j$ $u_1^2 + u_2^2 = 1$.
 $D_{\vec{u}}f(1,-1) = (3i - 4j) \cdot (u_1i + u_2j) = 3u_1 - 4u_2 = 4$

(e) $3u_1 - 4u_2 = -3$, $u_1 = \frac{4u_2 - 3}{3}$
 $\begin{cases} u_2 = 0, u_1 = -1 \\ u_2 = \frac{24}{25}, u_1 = \frac{7}{25} \end{cases}$

$\begin{cases} u_2 = \frac{2}{10} \\ u_1 = \frac{25}{9} \end{cases}$

Figure 5: Section 14.5, problem 24