Brief solutions to selected problems in homework 06

1. Section 14.2: Solutions, common mistakes and corrections:

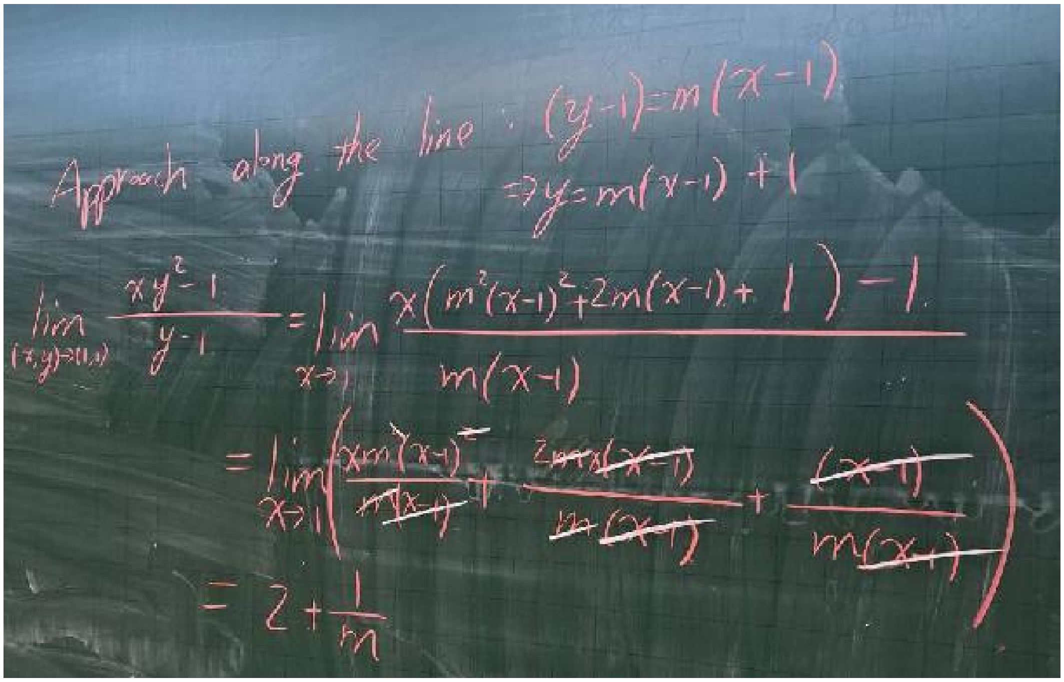

Figure 1: Section 14.2, problem 49

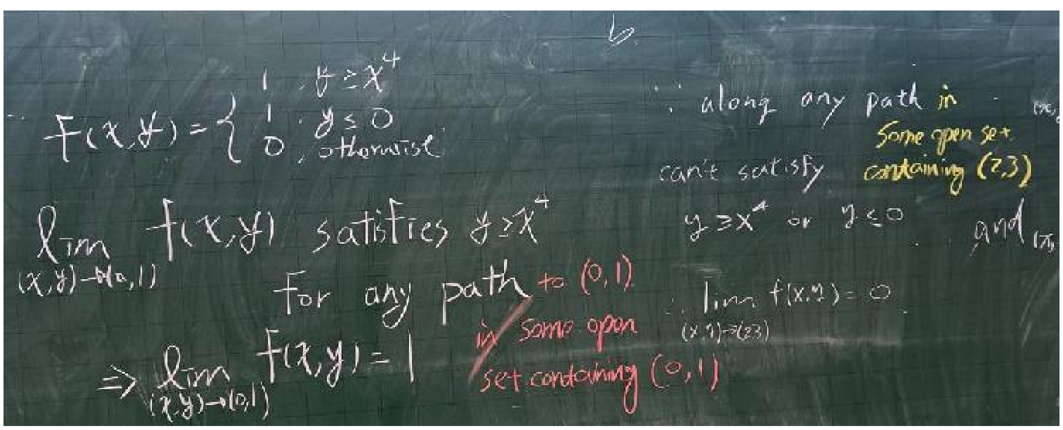

Figure 2: Section 14.2, problem 51

Remark: The Two Path Test can only be used to show the limit does not exist. If the two limits along two different paths are the same, it is not enough to say the 2D limit is the same as the limit along the two different paths.

To show the limit exists (and equals L), one needs to check that the values of $f(x, y)$ inside $0 < \sqrt{(x - x_0)^2 + (y - y_0)^2} < \delta$ to be close enough to L. The key here is to find an appropriate $\delta > 0$ for each $\varepsilon > 0$.

In problem (a), $\delta = \frac{1}{2}$ will work for any $\varepsilon > 0$ since

$$\{(x,y), \quad 0 < \sqrt{(x-0)^2 + (y-1)^2} < \frac{1}{2}\} \subset \{(x,y), y \geq x^4\}$$

Since if (x,y) is in the first set, then $|x| < \frac{1}{2}$, $y > 1 - \frac{1}{3}$, so $y > \frac{1}{2} > (\frac{1}{2})^4 > x^4$).
Similarly, it is easy to check that $\delta = 1$ (or smaller) works for problem (b).
For problem (c), one can apply the Two Path Test by checking the limits on the two paths $y = 2x^4$ and $y = \frac{1}{2}x^4$.

2. Section 14.3: Solutions, common mistakes and corrections:

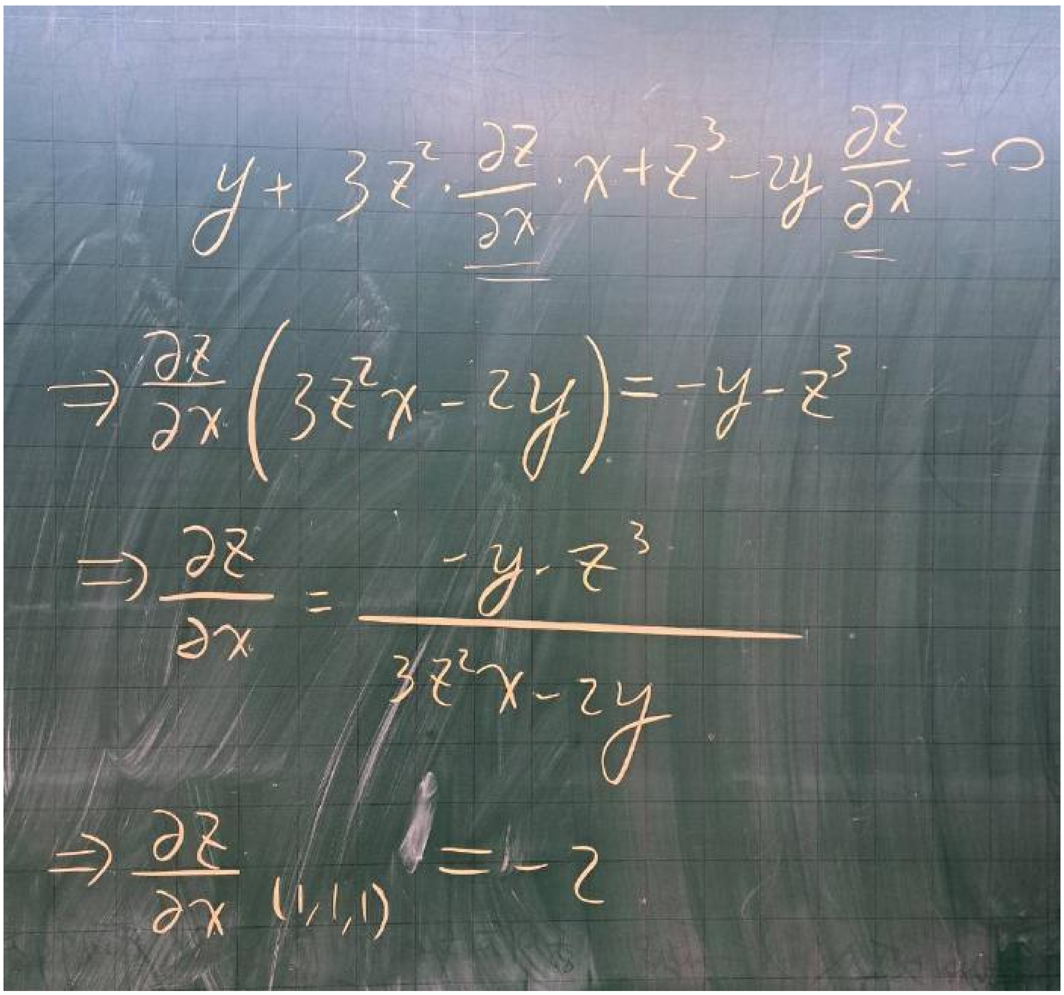

Figure 3: Section 14.3, problem 65