

## Brief solutions to selected problems in homework 05

### 1. Section 10.9: Solutions, common mistakes and corrections:

9

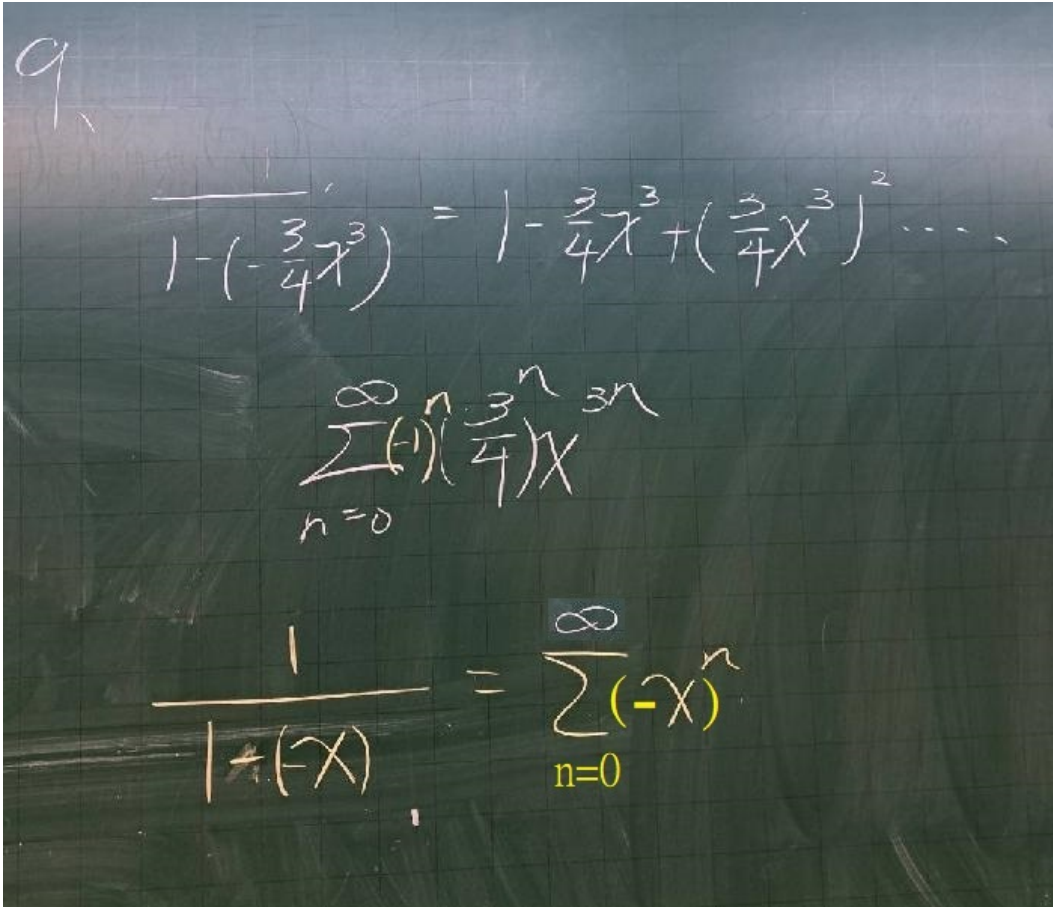
$$\frac{1}{1 - \left(-\frac{3}{4}x^3\right)} = 1 - \frac{3}{4}x^3 + \left(\frac{3}{4}x^3\right)^2 \dots$$
$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{4}\right)^n x^{3n}$$
$$\frac{1}{1 + (-x)} = \sum_{n=0}^{\infty} (-x)^n$$


Figure 1: Section 10.9, problem 9

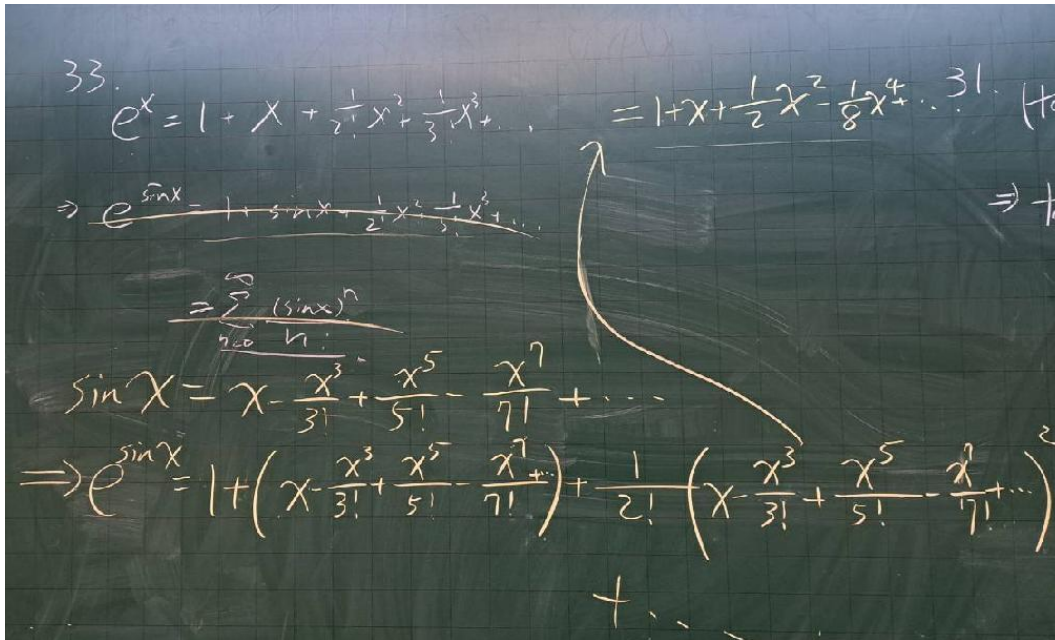


Figure 2: Section 10.9, problem 33

Remark:

$$x^2 \text{ term: } \frac{1}{2!} \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)^2 : \frac{1}{2}$$

$$x^3 \text{ term: } \frac{1}{1!} \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) + \frac{1}{3!} \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)^3 : -\frac{1}{3!} + \frac{1}{3!} = 0$$

$$x^4 \text{ term: } \frac{1}{2!} \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)^2 + \frac{1}{4!} \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)^4 : \frac{1}{2!} \cdot \frac{-2}{3!} + \frac{1}{4!} = -\frac{1}{8}$$

2. Section 10.10: Solutions, common mistakes and corrections:

$$\begin{aligned}
 [\tan^{-1} t]_x^\infty &= \frac{\pi}{2} - \tan^{-1} x \\
 &= \int_x^\infty \frac{1}{1+t^2} dt \\
 &= \int_x^\infty \left( \frac{1}{t^2} - \frac{1}{t^4} + \frac{1}{t^6} - \frac{1}{t^8} + \dots \right) dt \\
 &= \lim_{b \rightarrow \infty} \left( -\frac{1}{t} + \frac{1}{3t^3} - \frac{1}{5t^5} + \frac{1}{7t^7} - \dots \right) \Big|_x^\infty \\
 &= \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \dots
 \end{aligned}$$

Figure 3: Section 10.10, problem 66-1

$$\begin{aligned}
 \tan^{-1}(x) &= \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \dots, \quad x > 1 \\
 [\tan^{-1}(x)]_{+\infty}^x &= \frac{\pi}{2} + \tan^{-1}(x) \\
 &= \int_{-\infty}^x \frac{1}{1+t^2} dt \\
 &= -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \dots, \quad x < -1 \\
 \Rightarrow \text{When } x < -1, \tan^{-1}(x) &= \frac{-\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \dots
 \end{aligned}$$

Figure 4: Section 10.10, problem 66-2