

Brief solutions to selected problems in homework 03

1. Section 10.6: Solutions, common mistakes and corrections:

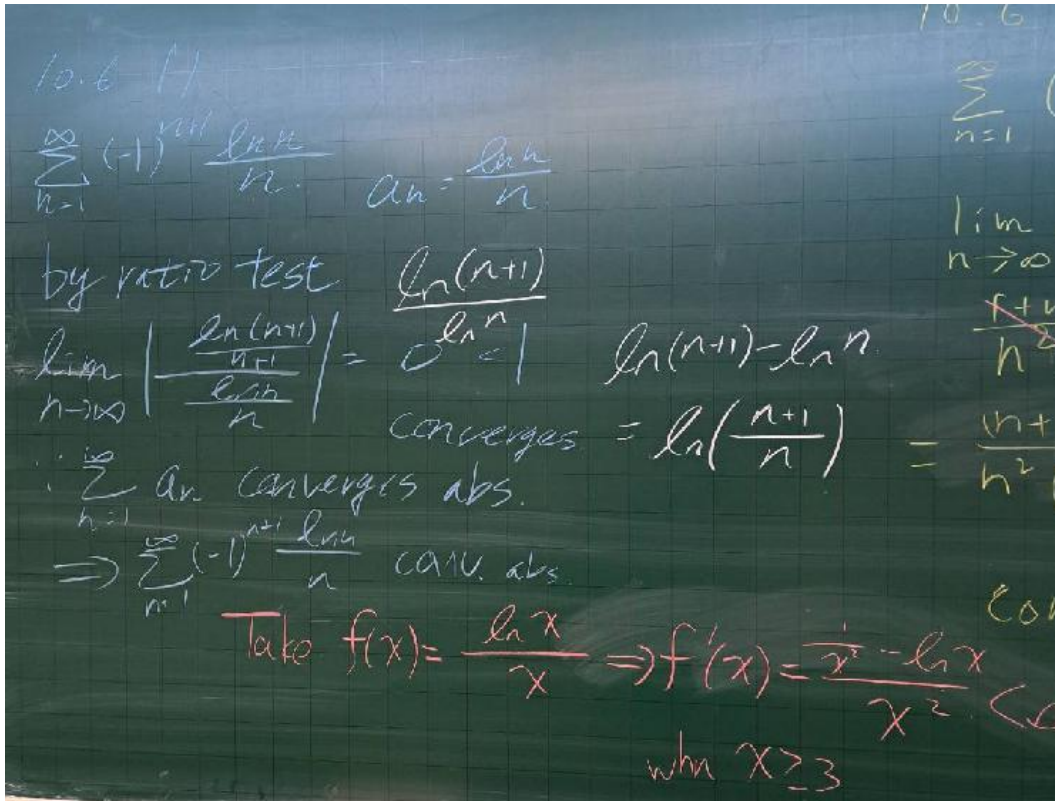


Figure 1: Section 10.6, problem 11

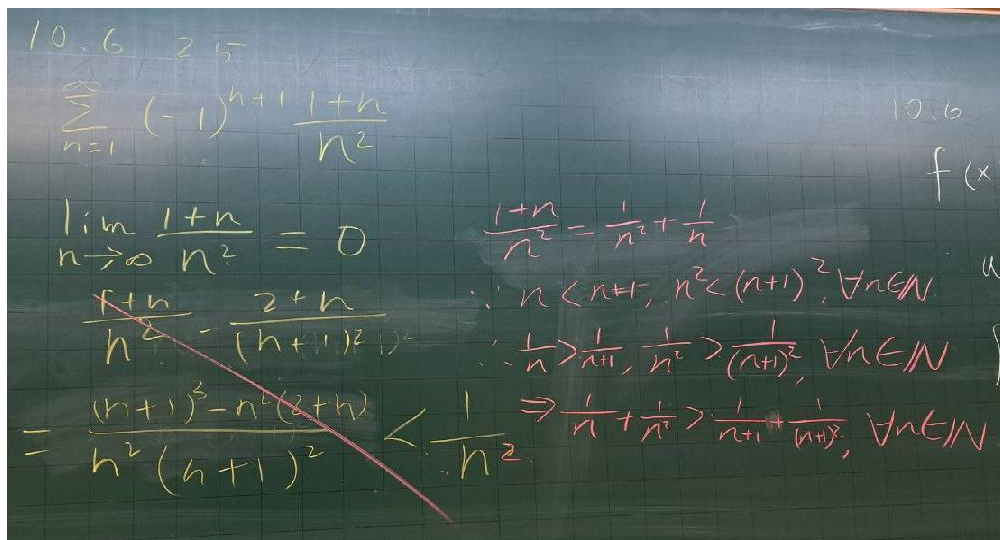


Figure 2: Section 10.6, problem 25. Method 2: Use Limit Comparison Test to compare  $|a_n|$  with  $\frac{1}{n}$

10.6 28

$$f(x) = \frac{1}{x \ln x}, \quad f'(x) = \frac{(\ln x + 1)}{(x \ln x)^2} < 0$$

$u_n > u_{n+1} > 0$  for  $n \geq 2$   
 $\Rightarrow$  series conv. by Alternating Series Test

$\int_2^{\infty} \frac{dx}{x \ln x} = \lim_{b \rightarrow \infty} \int_2^b \frac{(\frac{1}{x})}{\ln x} dx$

$= \lim_{b \rightarrow \infty} \ln(\ln x) \Big|_2^b = \infty$

$\therefore \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n \ln n}$  (divergent)

$\therefore \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}$  conv. but not absolutely

Figure 3: Section 10.6, problem 28

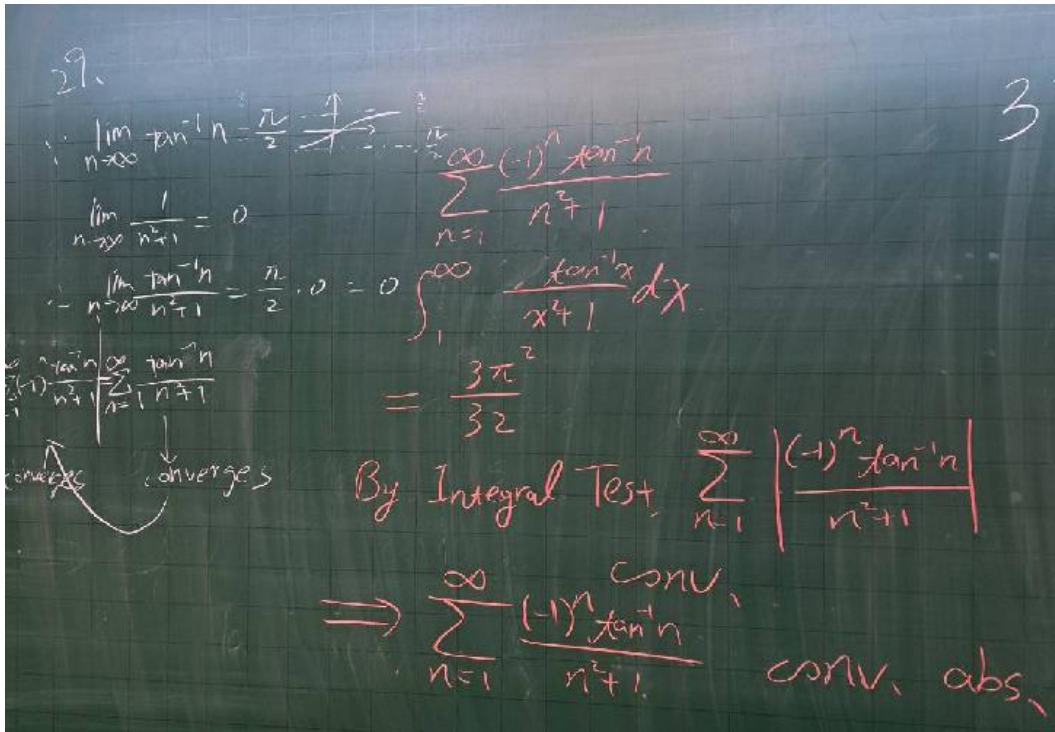


Figure 4: Section 10.6, problem 29. Remark:  $\int_{x=1}^{\infty} \frac{\tan^{-1} x}{1+x^2} dx = \int_{y=\frac{\pi}{4}}^{\frac{\pi}{2}} y dy$

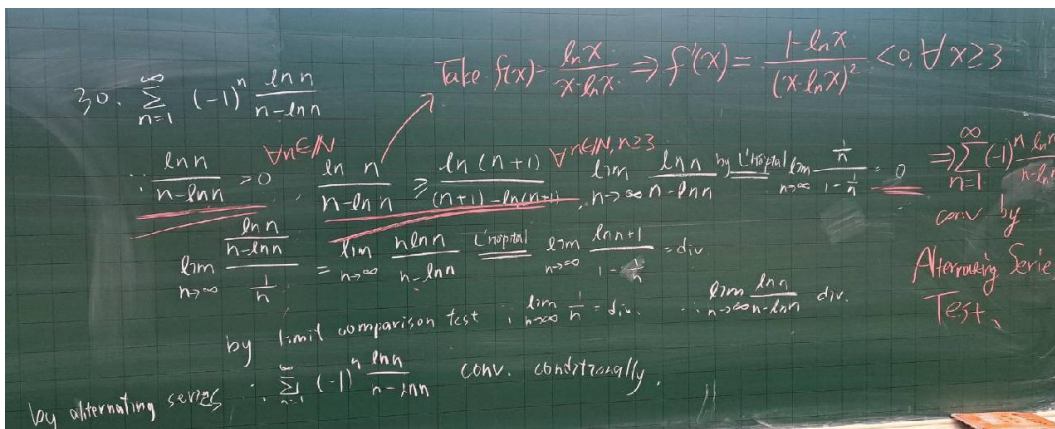


Figure 5: Section 10.6, problem 30



41.  $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$

$$\sqrt{n+1} - \sqrt{n} \times \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

and  $\left(\frac{1}{\sqrt{n+1} + \sqrt{n}}\right)$  is a decreasing sequence of positive terms which converges to 0  $\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1} + \sqrt{n}}$  converges

but  $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$  diverges by the Limit Comparison with  $\frac{1}{\sqrt{n}} \Rightarrow$  a divergent p-series

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n+1} + \sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n}} + 1} = \frac{1}{2} \Rightarrow$$

$\Rightarrow$  converges conditionally

Figure 6: Section 10.6, problem 41

49.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

$$|\text{error}| < \left| (-1)^{5+1} \frac{1}{5} \right| = 0.2$$

Figure 7: Section 10.6, problem 49

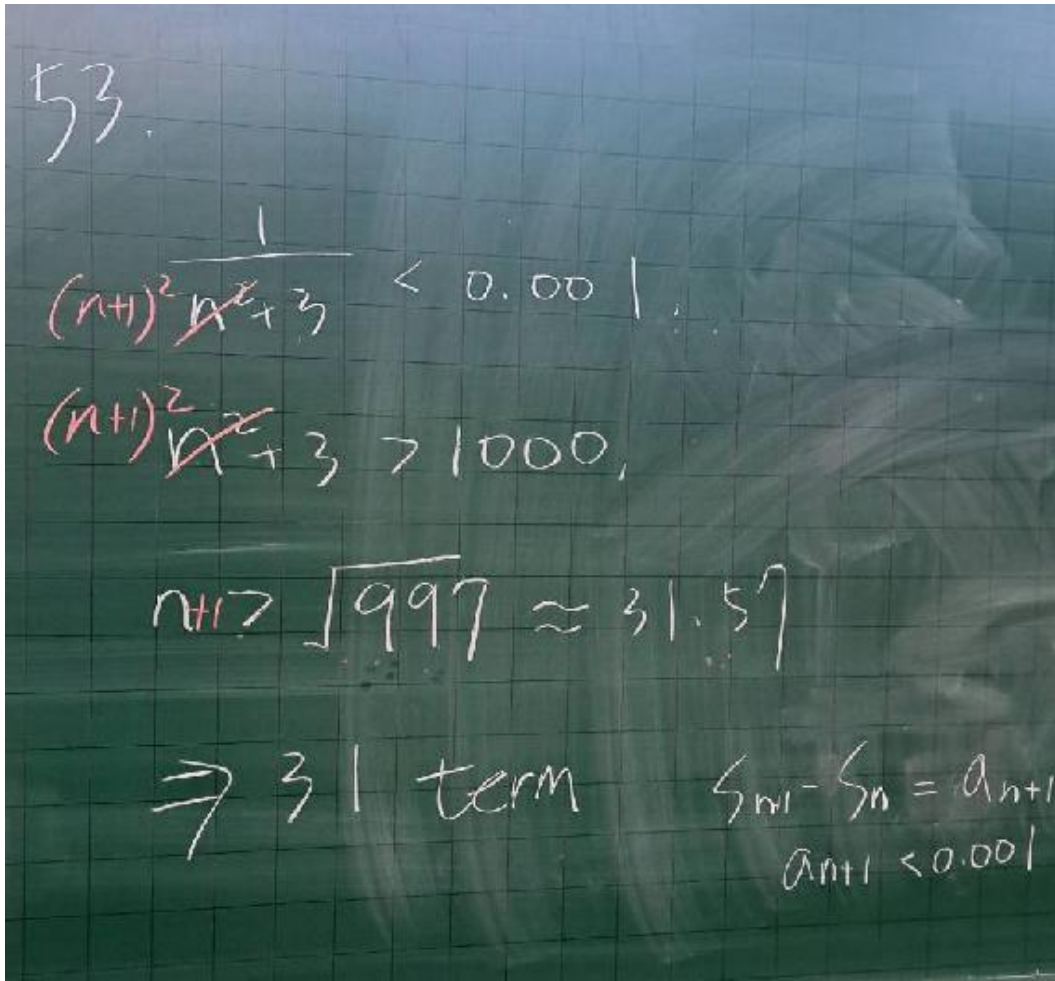


Figure 8: Section 10.6, problem 53. Note: the last few lines should read " $|S_n - S| < |a_{n+1}| < 0.001$ " instead.

2. Section 10.7: Solutions, common mistakes and corrections:

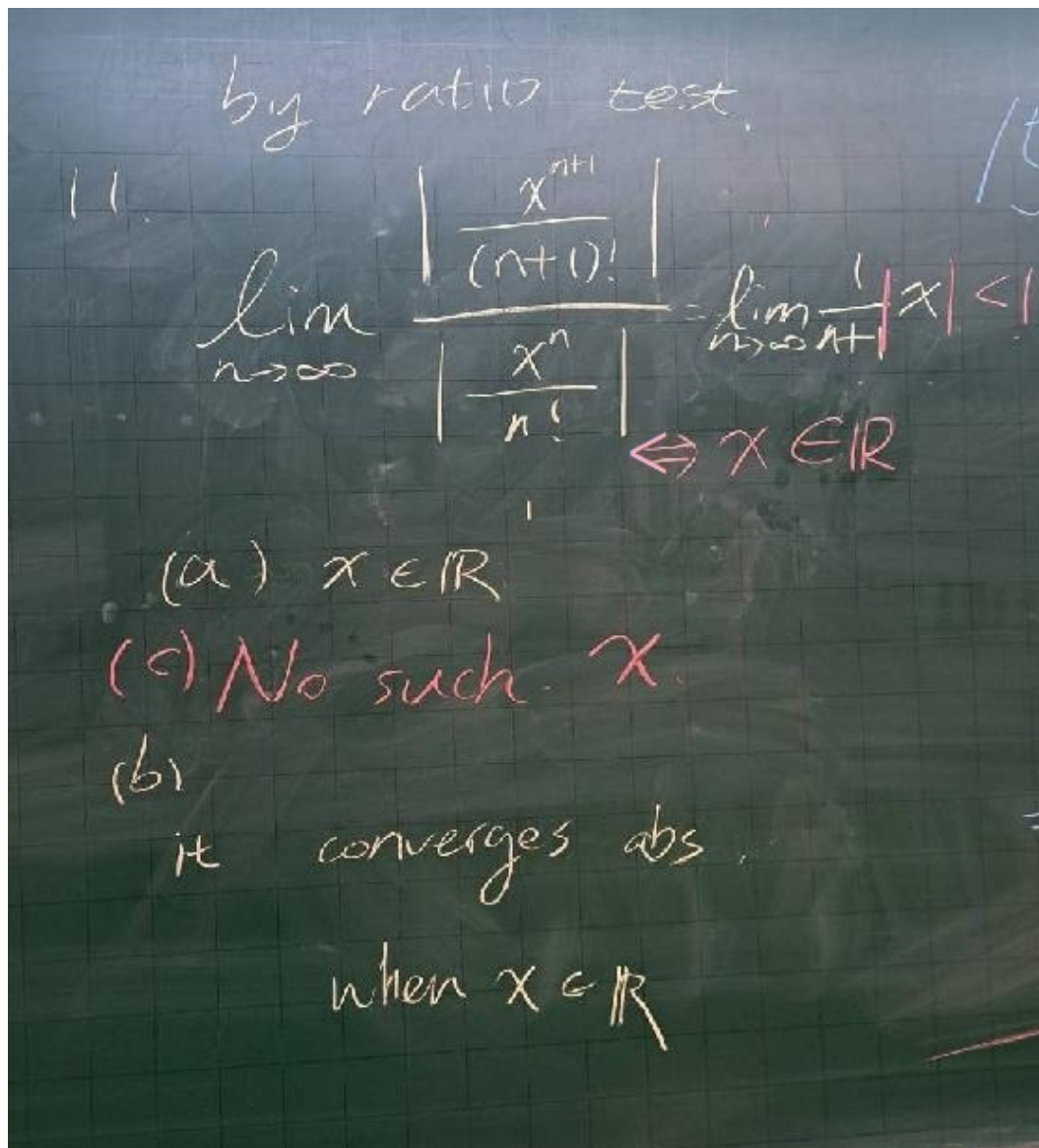


Figure 9: Section 10.7, problem 11



15.

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+3}} x^n$$

Root

$$\lim_{n \rightarrow \infty} \left| \frac{\frac{1}{\sqrt{n+3}}}{\frac{1}{\sqrt{n+3}}} \right| = \frac{\sqrt{(n+1)^2+3}}{\sqrt{n^2+3}} = 1 \Leftrightarrow |x| < 1 \Leftrightarrow -1 < x < 1$$

When  $x = -1$ ,  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+3}}$  conv. by Alternating Series Test

When  $x = 1$ ,  $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+3}}$  div.

(a) radi. 1, int. of conv.:  $[-1, 1)$   
 (b) int. of abs. conv.:  $(-1, 1)$   
 (c)  $x = -1$

so when  $|x| < 1$ ,  $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+3}} x^n$  conv.

Figure 10: Section 10.7, problem 15

23.

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n$$

Root

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) |x| = |x| < 1 \Leftrightarrow -1 < x < 1$$

(a) Radius = 1, int. of conv.:  $(-1, 1)$   
 (b) conv. (abs)  $(-1, 1)$   
 (c) No such  $x$ .

Alternating Series Test

When  $x = -1$ , by  $n$ th-Term Test,  $\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^n$  div.  
 When  $x = 1$ , by  $n$ th-Term Test,  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$  div.

Figure 11: Section 10.7, problem 23

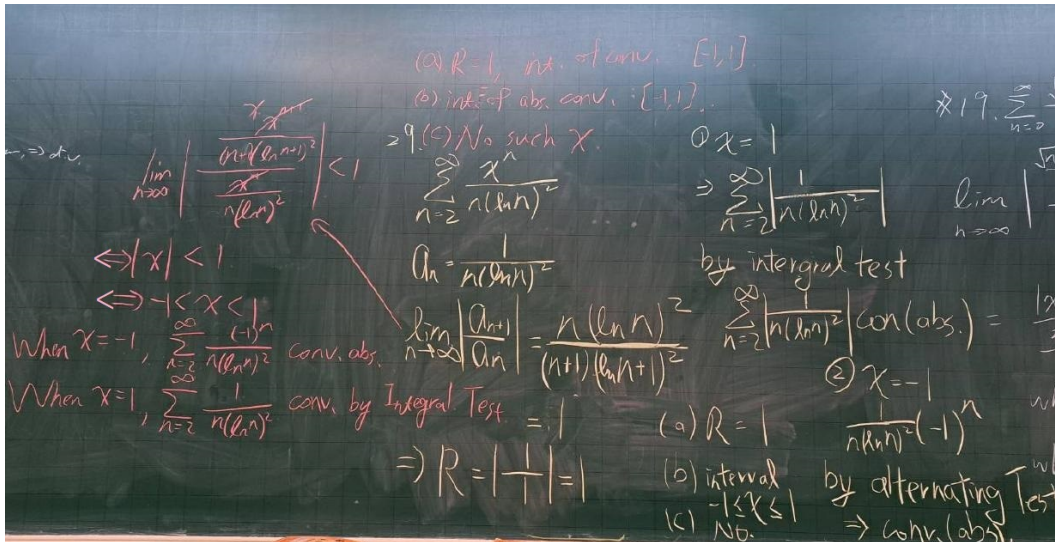


Figure 12: Section 10.7, problem 29

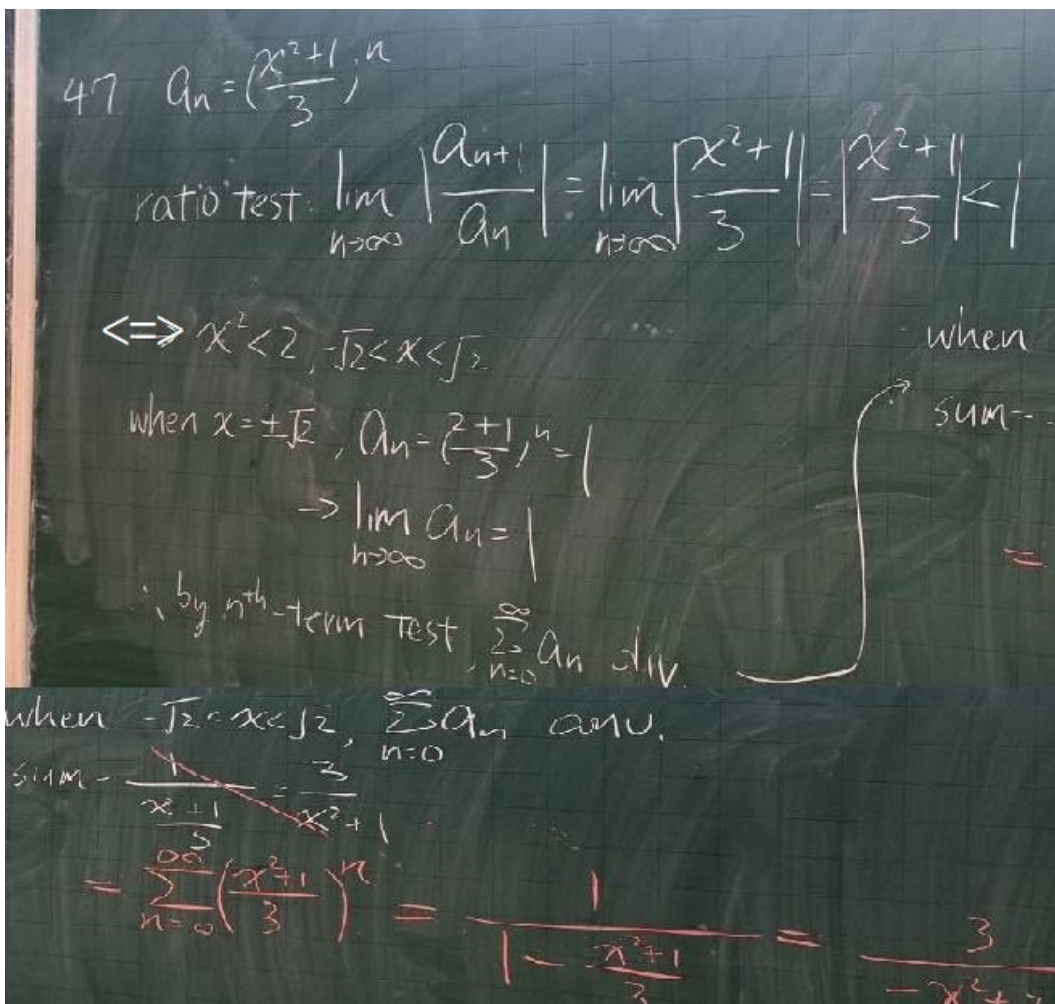


Figure 13: Section 10.7, problem 47