$Calculus \ II, \ Spring \ 2022 \ (Thomas' \ Calculus \ Early \ Transcendentals \ 13ed), \ http://www.math.nthu.edu.tw/~wangwc/$

Brief solutions to selected problems in homework 03

1. Section 10.6: Solutions, common mistakes and corrections:

by rotio test (n(n+i)) $\lim_{n \to \infty} \left| \frac{2n(n+i)}{n} \right|^{-2} \frac{2n^{n}}{n} \left| \frac{2n(n+i)}{n} \right|^{-2nn}$ $\lim_{n \to \infty} \left| \frac{2n(n+i)}{n} \right|^{-2nn} \frac{2n(n+i)}{n(n+i)} - 2nn$ $\lim_{n \to \infty} \left| \frac{2n(n+i)}{n} \right|^{-2nn} \frac{2n(n+i)}{n(n+i)} + 2nn(n+i)$ $\lim_{n \to \infty} \left| \frac{2n(n+i)}{n(n+i)} \right|^{-2nn(n+i)} \frac{2n(n+i)}{n(n+i)} + 2nn(n+i)$ $\lim_{n \to \infty} \left| \frac{2n(n+i)}{n(n+i)} \right|^{-2nn(n+i)} + 2nn(n+i)}$ Eo

Figure 1: Section 10.6, problem 11

n2 $\frac{1+n}{n^2} = 0 \qquad n^2 = n^2 t \frac{1}{n}$ $\frac{1+n}{n^2} = 0 \qquad n^2 = n^2 t \frac{1}{n}$ $\frac{1+n}{n^2} = 0 \qquad n^2 t \frac{1}{n^2} \frac{1}{n^2}$ $\lim_{n \to \infty} \frac{1+n}{n^2} = 0$ n+1)3-n-12+1-

Figure 2: Section 10.6, problem 25. Method 2: Use Limit Comparison Test to compare $|a_n|$ with $\frac{1}{n}$

XlnX Alternating Series Test $\int \frac{1}{\sqrt{2}} \frac{1}$ Un > 2 xenx dx ∞ $= \lim_{n \to \infty} \ln \left(\ln X \right) \Big|_{2}^{5}$ 00 5° $|a_n| = 2$ nlah n=1 N=1 (divergent) Con but not absolutely

Figure 3: Section 10.6, problem 28

tan h $=\frac{3\pi^{2}}{32}$ By Integral Test, $\sum_{n=1}^{\infty} \frac{(-1)^{n} \tan^{n}}{\pi^{2} + 1}$ $\sum_{n=1}^{\infty} \frac{(-1)^{n} \tan^{n}}{\pi^{2} + 1}$ anverges

Figure 4: Section 10.6, problem 29. Remark: $\int_{x=1}^{\infty} \frac{\tan^{-1} x}{1+x^2} dx = \int_{y=\frac{\pi}{4}}^{\frac{\pi}{2}} y dy$

Take $f(x) = \frac{f_n x}{x \cdot h_n x} \Rightarrow f'(x) = \frac{f - l_n x}{(x \cdot h_n x)^2} < 0, \forall x \ge 3$ 30. \$ (-1)ⁿ <u>lnn</u> Anel In n In (n+1) Vrell, n23 Inn my Lippellon - n n-0nn = (n+1)-laun=1, n-200 - Pnn - 100 1-1-Tim <u>hlnn</u> <u>(nopital</u> <u>lnm</u> <u>dnn+1</u> = diu = noo <u>h-lnn</u> <u>noo 1 - h</u> (comparison test i <u>hneo h</u> = diu Rim Ing div. by alternating sevies

Figure 5: Section 10.6, problem 30

41. 5 (-1)"(JATI-JA) Unti-JA X JATT + JA Until + JA and (JATT + JA) is a decreasing soquemee of positive terms which converges to 0=) S (-V) (anargos but S laid = S - duringes by O =) S (-V) (anargos hat S laid = S - duringes by the Libert Corporison with 1/ = a divergent p-series Run (JATI +JA) = Rim JATI +JA = Rim 1 100 (JATI +JA) = Rim JATI +JA = Rim 1 100 JI+++1 > converger conditionally

Figure 6: Section 10.6, problem 41



Figure 7: Section 10.6, problem 49

53.

$$(n+1)^{2}n^{2}n^{3} \le 0.001$$
.
 $(n+1)^{2}n^{2}n^{3} \ge 1000$.
 $n+12 \int 997 \approx 31.57$
 $7 31 term 5_{n} = 0.001$
 $ant1 \le 0.001$

Figure 8: Section 10.6, problem 53. Note: the last few lines should read " $|S_n - S| < |a_{n+1}| < 0.001$ " instead.

2. Section 10.7: Solutions, common mistakes and corrections:

(n+1)! (a) XER) No such (6) converges abs when x < 1R it

Figure 9: Section 10.7, problem 11

 $x=1, \frac{2}{h=0}, \frac{1}{N^{2}+3}, \frac{1$ TRAJ Mhere X X CONV

Figure 10: Section 10.7, problem 15

 $\sum_{n=1}^{\infty} (1+\frac{1}{n})^n \chi^n$ $\frac{t}{T} \lim_{n \to \infty} \frac{(1+\frac{1}{n})|\chi|}{T} = \frac{1}{2} |\chi| = \frac$ mating Series Test 15, Conv. (abs) (-1,1) (c) No such X. When X=1, by nth-Term Test, $\sum_{n=1}^{\infty} (-1)^n (1+\frac{1}{n})^n div.$ When X=1, by nth-Term Test, $\sum_{n=1}^{\infty} (1+\frac{1}{n})^n div.$

Figure 11: Section 10.7, problem 23

by intergral $n=2[n(\lambda n)^2]$ (In = n(9m/1) onv, by Integral =) R =

Figure 12: Section 10.7, problem 29

 $G_n = \left(\frac{x^2+1}{3}\right)^n$ ratio test $\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n \to \infty} \left|\frac{x^2+1}{3}\right| = \left|\frac{x^2+1}{3}\right|$ 47 <=> 1 ² < 2 j-J2 < X < j → when When x=+12, (An-(2+1))= > Im Qu= term test 230 when -Tze xejz, 29-Sum -

Figure 13: Section 10.7, problem 47