Homework 13

1. Section 16.3: Problems 1, 3, 5, 9, 21, 26, 29.

2. Section 16.3:
   Let $\boldsymbol{F} = \dfrac{x}{\sqrt{x^2+y^2}}\,\boldsymbol{i} + \dfrac{y}{\sqrt{x^2+y^2}}\,\boldsymbol{j} + 0\,\boldsymbol{k}$ and $\boldsymbol{G} = \dfrac{-y}{x^2+y^2}\,\boldsymbol{i} + \dfrac{x}{x^2+y^2}\,\boldsymbol{j} + 0\,\boldsymbol{k}$.

   (a) Show that both $\boldsymbol{F}$ and $\boldsymbol{G}$ satisfy the component test.

   (b) The natural domain of both $\boldsymbol{F}$ and $\boldsymbol{G}$ is $\{(x, y, z),\ x^2 + y^2 \neq 0\}$ (that is where $\boldsymbol{F}$ and $\boldsymbol{G}$ are defined). Show that $\boldsymbol{F}$ is conservative in this domain by finding its potential function.

   (c) Show that $\boldsymbol{G}$ is NOT conservative in this domain (see Example 5 on p990).

   (d) If given another $\boldsymbol{H}$ satisfying the component test in this domain, how do you determine whether $\boldsymbol{H}$ is conservative?

3. Section 16.3:
   Let $\boldsymbol{F} = \dfrac{x}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{i} + \dfrac{y}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{j} + \dfrac{z}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{k}$. What is the natural domain of $\boldsymbol{F}$? Show that $\boldsymbol{F}$ satisfies the component test in this domain. Is this domain simply connected? Is $\boldsymbol{F}$ conservative in this domain?

4. Section 16.4: Problems 10, 17, 19, 23, 27, 38, 39.

   Hints:

   In problem 17, a polar curve $r = f(\theta)$ can be parameterized by $x(\theta) = f(\theta)\cos\theta$, $y(\theta) = f(\theta)\sin\theta$.

   Problem 19 can be computed easier using Green's Theorem.

   On a circle $x^2 + y^2 = a^2$, it is a useful tip to remember that $\boldsymbol{n} = \dfrac{(x, y)}{a}$, $ds = a\,d\theta$ and $\boldsymbol{n}\,ds = (x, y)\,d\theta$.

5. Section 16.4: Verify Green's Theorem in tangential form and normal form for the vector field $\boldsymbol{F} = (M(x, y), 0)$ on R, where M and its partial derivatives are all continuous in R, the region illustrated in class. That is, R is bounded by $x = 0$, $y = 0$ and the curve $y = f(x)$, $0 \le x \le a$ with $f(0) = b$ and $f(a) = 0$, which at the same time can be described as $x = g(y)$, $0 \le y \le b$ with $g(0) = a$ and $g(b) = 0$. In the computation of the line integrals on the three portions of ∂R, pay attention to finding suitable parametrizations with correct orientation.