Homework 04

1. Section 10.7: 51, 55, 57, 60.

2. Section 10.7: Use the power series representation of $\dfrac{1}{1-x}$ to find the power series representation of $\ln(1-x)$ on $|x|<1$.

3. Section 10.7: Assuming the power series representation of

$$\frac{1-x^2+x^4-\cdots}{1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\cdots}$$

exists. Find the first three nonzero terms of this power series using the method in Lecture 07, page 3 (undetermined coefficients) or page 4 (long division).

4. Section 10.8: Problems 5 ($n=3$), 7 ($n=3$), 15, 23, 29, 35.

   Hint for problem 5:

   Method 1: Find $f^{(n)}(a)$ by repeated differentiation.

   Method 2: Write $x = x-2+2 = 2(1+\dfrac{x-2}{2})$ and use the formula for geometric series.

5. Section 10.8: Let

$$f(x) = \begin{cases} 0, & x = 0 \\ e^{-1/x^2}, & x \neq 0 \end{cases}$$

   It is known that $f^{(n)}(0) = 0$ for all n. Verify this for $f'(0)$ and $f''(0)$ (go through the details).

   Hint:

   Step 1: Compute $f'(0) = \lim_{x\to 0} \dfrac{f(x)-f(0)}{x-0} = 0$ (see page 12 of Lecture 08).

   Step 2: Compute $f''(0) = \lim_{x\to 0} \dfrac{f'(x)-f('0)}{x-0}$. Since we already computed $f'(0) = 0$ from step 1, all we need is to compute $f'(x)$ for $x \neq 0$ to find this limit.

   If $x \neq 0$, then we can compute directly $f'(x) = \dfrac{d}{dx}e^{-1/x^2} = \cdots$. Evaluation of the limit in step 2 is similar to the process in step 1 (use the trick given on page 12 of Lecture 08).