Homework 02

1. Section 10.3: Problems 7, 27, 28, 31, 33, 37, 41, 55.

   Hint for problem 41: If both $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n \pm b_n)$ could either converge or diverge in general.

   To determine the convergence or divergence, one can, for example, simplify $a_n = \dfrac{a}{n+2} - \dfrac{1}{n+4} = \dfrac{\cdots}{(n+2)(n+4)}$ and use one of the comparison methods in Section 10.4.

2. Section 10.4: Problems 15, 16, 17, 27, 29, 31, 43, 45, 51, 61, 62.

   Hints for problem 61:

   (a) It suffices to consider the case $q > 0$. The case $q \leq 0$ is much easier. (Why?)

   (b) If you find it difficult to deal with general $p > 1$ and $q > 0$, try to start with, for example, fixed $p = 1.5$, $q = 3$, and proceed with $r = (1+p)/2$. Then repeat your methods to general $p > 1$, $q > 0$.

   (c) Instead of evaluating $\lim_{n\to\infty} \dfrac{(\ln n)^q}{n^{p-r}}$, $q > 0$, it is enough (and easier) to evaluate $\lim_{n\to\infty} \dfrac{\ln n}{n^{\frac{p-r}{q}}}$.

   Hint in problem 62: It suffices to consider the case $q < 0$. The case $q \geq 0$ is much easier. $r = (1+p)/2$ also works in this case.

3. Section 10.5: Odd numbered problems in problem 17-43, 61, 65.

   Hint: In general, the ratio test applies if you see the factorial $(\cdots)!$ appearing in a_n. We did not have time for more examples in lecture 04. Do your best as time permits. We will proceed with more examples (and the proof) for both tests in lecture 05.

   Hint for problem 61: multiply by $2 * 4 * \cdots * 2n = 2^n n!$ both on the denominator and the numerator.

   Hint for problem 65: Ratio test will be inconclusive, but root test works.