

Brief solutions to Quiz 10

Jan 02, 2024:

1. (30 pts) Evaluate $\int_0^1 \tanh\left(\frac{t^2}{2}\right) t dt$.

Ans:

$$= \int_0^1 \tanh\left(\frac{t^2}{2}\right) d\left(\frac{t^2}{2}\right) = \int_0^1 \frac{\sinh\left(\frac{t^2}{2}\right)}{\cosh\left(\frac{t^2}{2}\right)} d\left(\frac{t^2}{2}\right) = \int_0^1 \frac{d\left(\cosh\left(\frac{t^2}{2}\right)\right)}{\cosh\left(\frac{t^2}{2}\right)} = \ln \cosh\left(\frac{t^2}{2}\right) \Big|_0^1 = \ln \cosh \frac{1}{2}$$

2. (30 pts) Order the functions $n \ln n$, $n\sqrt{n}$, $e^{0.1n}$ by rate of growth as $n \rightarrow \infty$. For simplicity of expression, you can use the notations $a \ll b$ to say b grows faster than a , and $a \approx b$ to say a and b grow at the same rate.

Ans:

$$n \ln n \ll n\sqrt{n} \ll e^{0.1n}$$

3. (40 pts) Evaluate $\int e^x \cos x dx$.

Ans:

See page 478 of the textbook, or page 8 of Lecture 26.