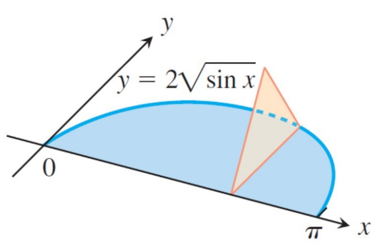
Brief solutions to Quiz 7

Nov 28, 2023:

1. (30 pts) Find the volume of the solid whose base is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross-sections perpendicular to the x-axis are equilateral triangles with bases running from the x-axis to the curve as shown in the figure:



Ans:

$$V = \int_0^\pi A(x)\,dx = \int_0^\pi \frac{1}{2} 2\sqrt{\sin x}\sqrt{3}\sqrt{\sin x}\,dx = \int_0^\pi \sqrt{3}\sin x\,dx = 2\sqrt{3}$$

2. (50 pts) Use any method (disks/washers or cylindrical shells) to find the volume of revolution obtained by rotating the region enclosed by $\{x = 4\}$, $\{y = 0\}$ and $\{y = \sqrt{x}\}$ around the x- axis and the y- axis, respectively.

Ans:

Method of disks/washers:

$$V_{x-\text{axis}} = \int_0^4 \pi\big((\sqrt{x})^2 - 0^2\big)dx = 8\pi$$

$$V_{y-\text{axis}} = \int_0^2 \pi\big(4^2 - (y^2)^2\big)dy = \frac{128}{5}\pi$$

Method of cylindrical shells:

$$V_{x-\text{axis}} = \int_0^2 2\pi y\big(4 - y^2\big)dy = 8\pi$$

$$V_{y-\text{axis}} = \int_0^4 2\pi x\big(\sqrt{x} - 0\big)dx = \frac{128}{5}\pi$$

3. (20 pts) Find the length of the curve $y = f(x) = \int_0^x \sqrt{\cos 2t}\, dt$ from $x = 0$ to $x = \frac{\pi}{4}$.

Ans:

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + (f'(x))^2}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 2x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x}\, dx = \sqrt{2} \int_0^{\frac{\pi}{4}} \cos x\, dx = 1$$