Calculus I, Fall 2023

# Brief solutions to Quiz 4

Nov 07, 2023:

1. (40 pts) Evaluate the derivative of $(\sin x)^{\tan x} + x^{\log_2(x)}$, where $0 < x < \dfrac{\pi}{2}$.

   **Ans**:

   Start with

   $$(\sin x)^{\tan x} + x^{\log_2(x)} = (e^{\ln(\sin x)})^{\tan x} + (e^{\ln x})^{\log_2 x} = e^{\tan x \ln(\sin x)} + e^{\ln x \log_2 x} = e^{\tan x \ln(\sin x)} + e^{\frac{\ln^2 x}{ln2}}$$

   and proceed with The Chain Rule:

   $$(e^{f(x)} + e^{g(x)})' = e^{f(x)} f'(x) + e^{g(x)} g'(x).$$

   A common mistake:

   $$\ln(F(x) + G(x)) = \ln F(x) + \ln G(x), \qquad Wrong!$$

2. (42 pts) Write down domains and ranges of all six inverse trigonometric functions and their derivatives (need not derive them).

   **Ans**:

   Domains and ranges: See page 62 and page 201 of the textbook.

   Derivatives: See page 205 of the textbook.

   See also Lecture 10 note for all the answers.

3. (18 pts) Use the linear approximation formula for $(1 + x)^k$ near $x = 0$ to get an approximate value of $\dfrac{1}{\sqrt{3.99}}$.

   **Ans**:

   $$\frac{1}{\sqrt{0.99}} = (4 - 0.01)^{\frac{-1}{4}} = \big(4(1 - \frac{0.01}{2})^{\frac{-1}{2}}\big) \approx \frac{1}{2}\big(1 - \frac{0.01}{4}(\frac{-1}{2})\big) = 0.500625$$