Calculus I, Fall 2023

Brief solutions to Quiz 1

Sep 19, 2023:

1. (30 pts) Evaluate $\lim_{x\to 0} \dfrac{\sqrt[3]{1+2x}-1}{x}$.

   Ans:

   $$\lim_{x\to 0} \frac{\sqrt[3]{1+2x}-1}{x} = \lim_{x\to 0} \left(\frac{\sqrt[3]{1+2x}-1}{x} \cdot \frac{\sqrt[3]{1+2x}^2 + \sqrt[3]{1+2x}+1}{\sqrt[3]{1+2x}^2 + \sqrt[3]{1+2x}+1} \right)$$

   $$= \lim_{x\to 0} \frac{(1+2x)-1}{x(\sqrt[3]{1+2x}^2 + \sqrt[3]{1+2x}+1)} = \lim_{x\to 0} \frac{2}{\sqrt[3]{1+2x}^2 + \sqrt[3]{1+2x}+1} = \frac{2}{3}.$$

2. (35 pts) If $\lim_{x\to 1} \dfrac{f(x)-2}{x-1} = 3$, find $\lim_{x\to 1} f(x)$.

   Ans:

   $\lim_{x\to 1} f(x) = 2$. See homework_01_s2.2_sln_mse23f.pdf for details.

3. (35 pts) Find $\lim_{x\to 0} x^2 \sin\dfrac{1}{x}$ and write down the full statement of the Theorem you use.

   Ans:

   Since $-x^2 \le x^2 \sin\dfrac{1}{x} \le x^2$ and $\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$, it follows from The Sandwich Theorem that $\lim_{x\to 0} x^2 \sin\dfrac{1}{x} = 0$.

   See Theorem 4, p86 for the statement of The Sandwich Theorem.