Brief solutions to selected problems in homework 13

1. Section 6.4: Solutions, common mistakes and corrections:

23. $\int 2\pi y\, ds = \int 2\pi y \sqrt{dx^2+dy^2} = \int 2\pi y \sqrt{\left(\frac{dx}{dy}\right)^2+1}\, dy$

$$\Rightarrow S = \int_{y=1}^{y=2} 2\pi y \sqrt{(x')^2+1}\, dy$$

$$= \int_{y=1}^{y=2} 2\pi y \sqrt{\left(y^3-\frac{1}{4y^3}\right)^2+1}\, dy \quad \left(x' = y^3 - \frac{1}{4y^3}\right)$$

$$= \int_1^2 2\pi y \sqrt{\left(y^3+\frac{1}{4y^3}\right)^2}\, dy$$

$$= \int_1^2 2\pi y \left[y^3+\frac{1}{4y^3}\right] dy \quad \left(\because y^3+\frac{1}{4y^3} \geq 0 \text{ if } y \in [1,2]\right)$$

$$= 2\pi\left(\frac{y^5}{5} - \frac{1}{4y}\Big|_1^2\right) = \frac{253}{20}\pi$$

Figure 1: Solution to Section 6.4, problem 23

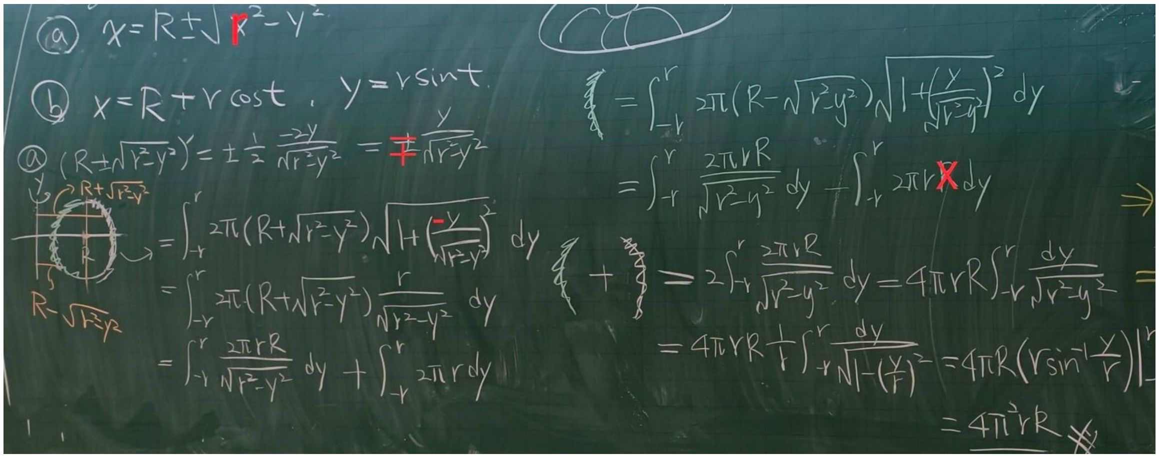

Figure 2: Solution to homework 13, problem 2(a)

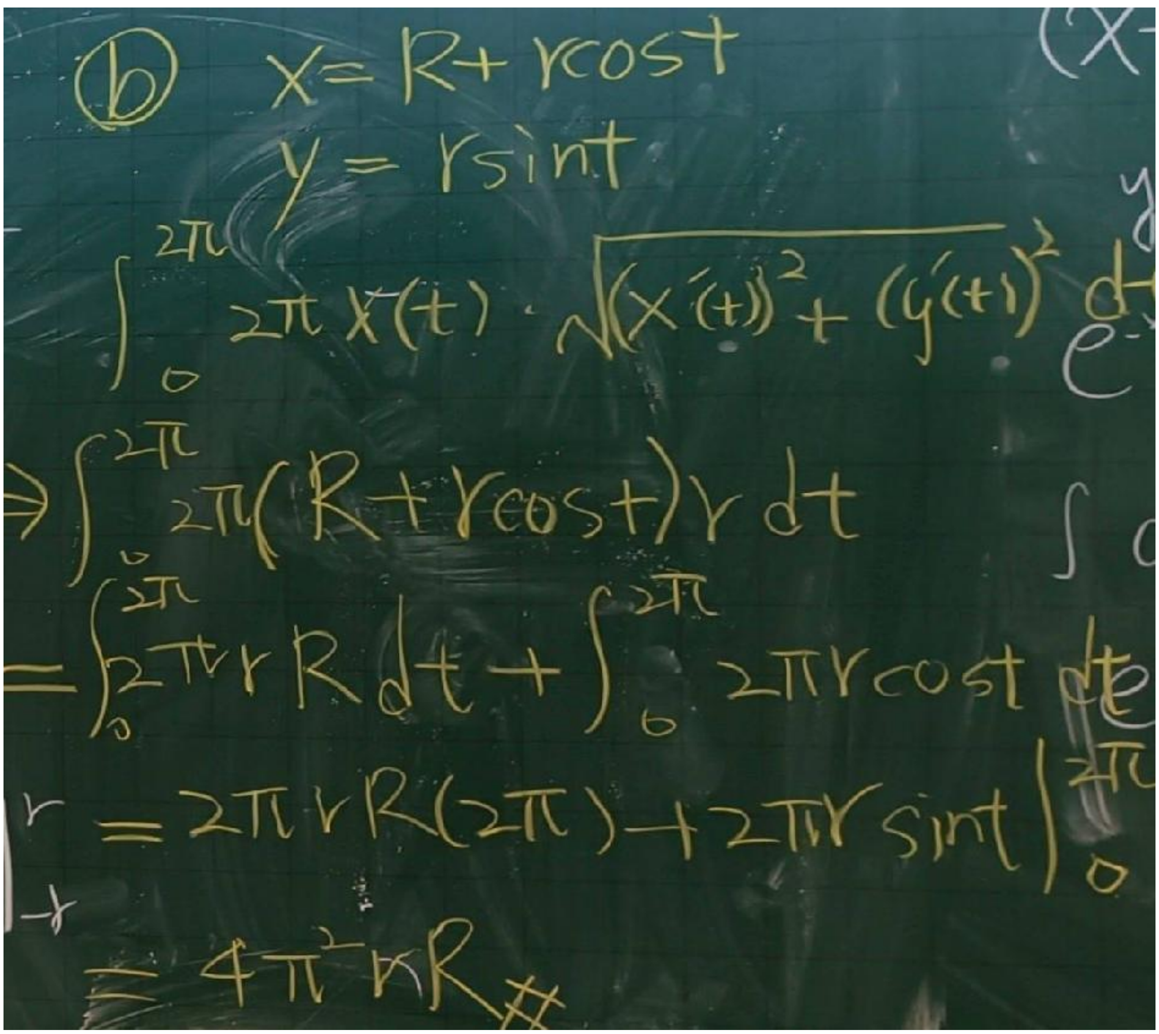

Figure 3: Solution to homework 13, problem 2(b)